# University of Jordan
# School of Engineering
# Electrical Engineering Department


# EE 204
# Electrical Engineering Lab

# EXPERIMENT 6
# INDUCTIVE REACTANCE

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# EXPERIMENT 6
# INDUCTIVE REACTANCE

## OBJECTIVE

Inductive reactance will be examined in this experiment. In particular, its relationship to the AC source frequency will be investigated, including a plot of inductive reactance versus frequency. In addition, AC power and power factor calculations will be introduced conducted.

## DISCUSSION

### Impedance, Reactance, Admittance and Susceptance

The AC current-voltage characteristic of an inductor, known as inductive *reactance* $X_L$, is directly proportional to frequency. Hence, the *impedance* of an inductor is:

$$\boldsymbol{Z_L} = jX_L = j\omega L$$

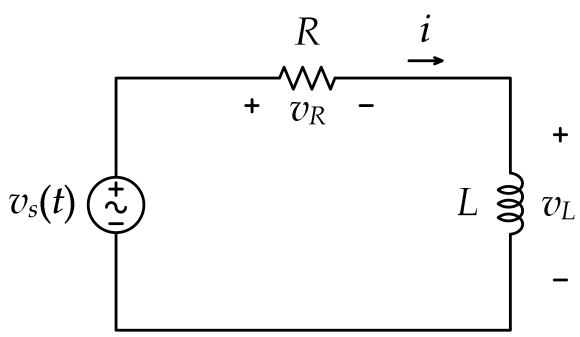
The inverse of the impedance is *admittance* $\boldsymbol{Y} = 1/\boldsymbol{Z} = G + jB$, which has the real part $G$ called *conductance*, and the imaginary part $B$ called *susceptance*. All three quantities have units of Siemens (S).

The inductive reactance may be determined experimentally by applying a known AC voltage across the inductor, measuring the resulting current, and dividing the two. This process may be repeated across a range of frequencies in order to obtain a plot of inductive reactance versus frequency.

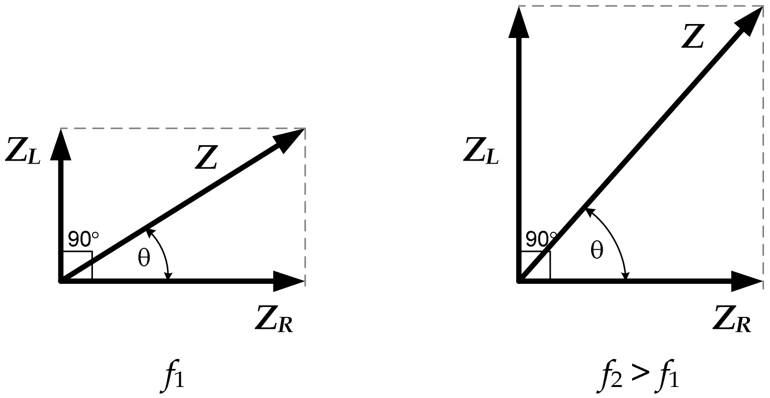
### AC-excited series RL circuit

For the series RL circuit shown below, the total impedance seen by the AC source is given by:

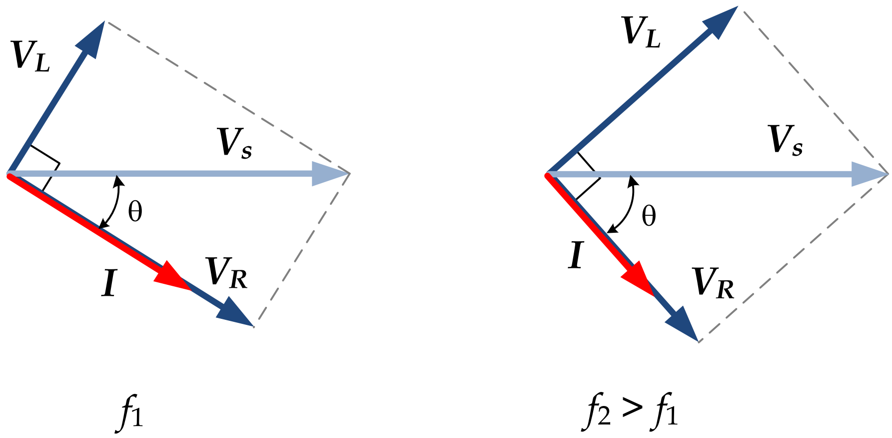
$$\boldsymbol{Z} = \boldsymbol{Z_R} + \boldsymbol{Z_L} = R + j\omega L = \sqrt{R^2 + (\omega L)^2} \measuredangle \tan^{-1}\left(\frac{\omega L}{R}\right)$$




Since the inductor impedance $\boldsymbol{Z_L}$ changes with the frequency of the source, the total impedance $\boldsymbol{Z}$ changes with frequency as well, as shown in the following phasor diagram. Notice how the resistor impedance remains constant with frequency.



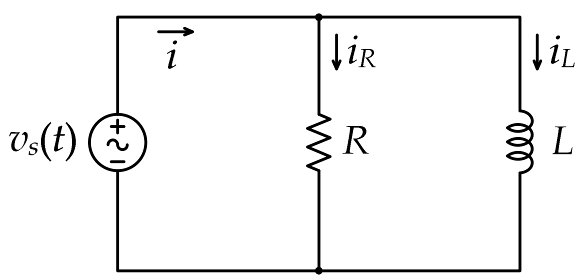

The following phasor diagram shows how the above impedance change affects the inductor voltage $\boldsymbol{V_L}$ and resistor voltage $\boldsymbol{V_R}$, both of which now change with frequency for a constant $\boldsymbol{V_S}$ due to the voltage divider rule. The current $\boldsymbol{I}$ also changes along with its phase shift compared to the source voltage $\boldsymbol{V_S}$, but the current remains lagging compared to the source voltage. Notice that complex numbers are added like vectors, not like scalars.



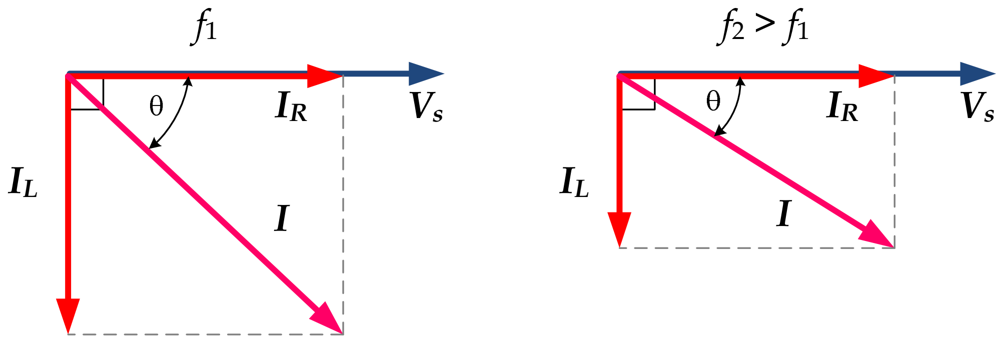

**AC-excited parallel RL circuit**

For the parallel RL circuit shown below, the total admittance seen by the AC source is given by:

$$\boldsymbol{Y} = \boldsymbol{Y_R} + \boldsymbol{Y_L} = \frac{1}{R} + \frac{1}{j\omega L} = \frac{1}{R} - j\frac{1}{\omega L} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L}\right)^2} \measuredangle - \tan^{-1}\left(\frac{R}{\omega L}\right)$$




The following phasor diagram shows how the inductive admittance change with frequency affects the inductor current $\boldsymbol{I_L}$, which decreases with increasing frequency for a constant source voltage $\boldsymbol{V_S}$. The resistance current $\boldsymbol{I_R}$, however, stays constant, which means that the total current $\boldsymbol{I}$ changes (due to phasor addition) along with its phase shift compared to the source voltage $\boldsymbol{V_S}$. Of course, the current lags the source voltage due to the inductive load.




**Power and Power Factor**

The average *complex power* $\boldsymbol{S}$ (units of VA) is given by the combination of the average *real power* $P$ and the average *reactive power* $Q$ as follows:

$$\boldsymbol{S} = P + jQ = V_{rms} I_{rms} \measuredangle(\measuredangle V - \measuredangle I) = \frac{1}{2} V_p I_p \measuredangle(\measuredangle V - \measuredangle I)$$

Hence, the real power $P$ (units of W) is given by:

$$P = V_{rms} I_{rms} \cos(\measuredangle V - \measuredangle I) = \frac{1}{2} V_p I_p \cos(\measuredangle V - \measuredangle I)$$

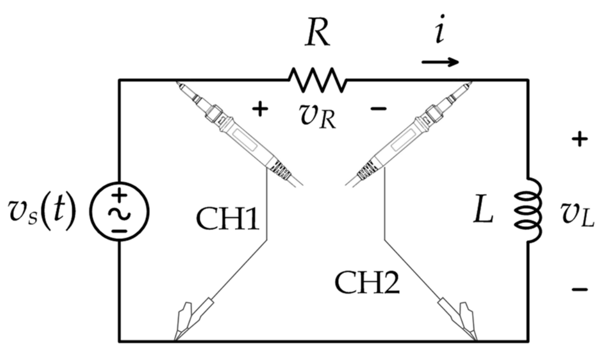
And the reactive power $Q$ (units of VAR) is given by:

$$Q = V_{rms} I_{rms} \sin(\measuredangle V - \measuredangle I) = \frac{1}{2} V_p I_p \sin(\measuredangle V - \measuredangle I)$$

where the $\cos(\measuredangle V - \measuredangle I)$ quantity is known as the *power factor* (PF), which is lagging for inductive loads, and leading for capacitive loads. Finally the *apparent power* is given by $|\boldsymbol{S}| = V_{rms} I_{rms} = 0.5 \times V_p \times I_p$.

**PROCEDURE A – AC-EXCITED SERIES RL CIRCUIT**

1. Construct the circuit shown below. Use $R$ = 820 Ω and $L$ = 10 mH.




2. Set the function generator to produce a sinusoidal waveform (AC) with frequency of 700 Hz, and *peak voltage* of $V_p = 3$ V.

**CAUTION:** Some older function generators have a defect and produce an AC signal with a slight DC shift (positive or negative). Hence, if you do not see a symmetric sinusoidal signal above and below zero volts, adjust the DC offset knob slightly to force a zero DC offset in the function generator output.

3. Use theoretical analysis to determine the voltages $\boldsymbol{V_L}$ and $\boldsymbol{V_R}$ and current $\boldsymbol{I}$ in the circuit at the different frequencies in Tables 1 and 2. Make sure to evaluate both magnitude and phase for each complex quantity, and record the expected period $T$ of the signals in milliseconds. Record the answers in the two tables? **Note**: Using MATLAB can quickly give you the theoretical answers if you define a vector of frequencies and then use array arithmetic.

4. Use the oscilloscope to measure the peak values of the voltages $\boldsymbol{V_S}$ and $\boldsymbol{V_L}$ and the phase shift of $\boldsymbol{V_L}$ compared to $\boldsymbol{V_S}$**.** Remember that you can change the horizontal sweep setting of the oscilloscope to make more accurate measurements of the phase. Also measure the period $T$ in milliseconds of the source voltage $\boldsymbol{V_S}$. Record the measurements in Table 1. You can use the "Automatic Measurement" feature of the oscilloscope explained below to speed up your procedure.

**CAUTION:** Whenever you change the frequency of the function generator, verify the period of the signal from the oscilloscope to get accurate readings. Also re-check the peak-to-peak voltage as the function generator might change the amplitude when you change the frequency.

5. Use the oscilloscope to measure the peak value of the voltage $\boldsymbol{V_R}$ and its phase shift compared to $\boldsymbol{V_S}$**.** Also measure the period $T$ in milliseconds of $\boldsymbol{V_R}$. Record the measurements in Table 2. You can use the "Automatic Measurement" feature of the oscilloscope to speed up your procedure.

**CAUTION:** Do *not* put CH2 of the oscilloscope across the resistor $R$ while simultaneously measuring $\boldsymbol{V_S}$ using CH1 since this will short circuit the inductor. Rather you can use one of the following tricks: (*a*) you can swap the locations of $L$ and $R$ in the circuit while keeping the oscilloscope connections unchanged, allowing you to measure $\boldsymbol{V_R}$, or alternatively (*b*) you can use the **A** – **B** mode in the oscilloscope, which subtracts the instantaneous signal on CH1 minus the signal on CH2 and displays the result, which is the instantaneous voltage across the resistor.

6. Which of the above two methods did you decide to use to measure $\boldsymbol{V_R}$?

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

7. Can we just subtract the magnitudes of $|\boldsymbol{V_S}| - |\boldsymbol{V_L}|$ to obtain the magnitude $|\boldsymbol{V_R}|$? Why or why not?

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

8. Now evaluate the *measured* current $\boldsymbol{I}$ and its phase shift compared to $\boldsymbol{V_S}$ by applying Ohm's law to the resistor using the measured value of $\boldsymbol{V_R}$ (i.e., $\boldsymbol{I} = \boldsymbol{V_R}/R$). Record the answer in Table 2.

9. What is the relationship between the periods $T$ of the two signals $\boldsymbol{V_S}$ and $\boldsymbol{V_R}$?

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

**Table 1**

| AC Source Frequency (Hz) | $V_S$ (peak) (V) | | $V_S$ period $T$ (ms) | | $V_L$ (peak) (V) | | $\measuredangle V_L$ with $V_s$ (Lead = positive) | |
|---|---|---|---|---|---|---|---|---|
| | Theory | Meas. | Theory | Meas. | Theory | Meas. | Theory | Meas. |
| 700 | 3 | | | | | | | |
| 1300 | 3 | | | | | | | |
| 2600 | 3 | | | | | | | |
| 5200 | 3 | | | | | | | |
| 9100 | 3 | | | | | | | |
| 14400 | 3 | | | | | | | |
| 22200 | 3 | | | | | | | |
| 39200 | 3 | | | | | | | |

**Table 2**

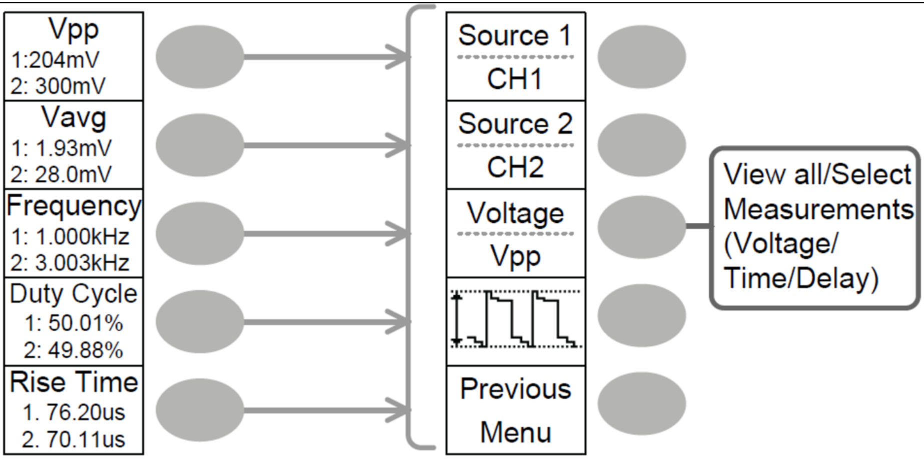
| AC Source Frequency (Hz) | $V_R$ (peak) (V) | | $\measuredangle V_R$ with $V_s$ (Lag = negative) | | $V_R$ period $T$ (ms) | | $I$ and $\measuredangle I$ (mA) $= V_R/R$ | |
|---|---|---|---|---|---|---|---|---|
| | Theory | Meas. | Theory | Meas. | Theory | Meas. | Theory | Meas. |
| 700 | | | | | | | | |
| 1300 | | | | | | | | |
| 2600 | | | | | | | | |
| 5200 | | | | | | | | |
| 9100 | | | | | | | | |
| 14400 | | | | | | | | |
| 22200 | | | | | | | | |
| 39200 | | | | | | | | |

**OSCILLOSCOPE AUTOMATIC MEASUREMENT**

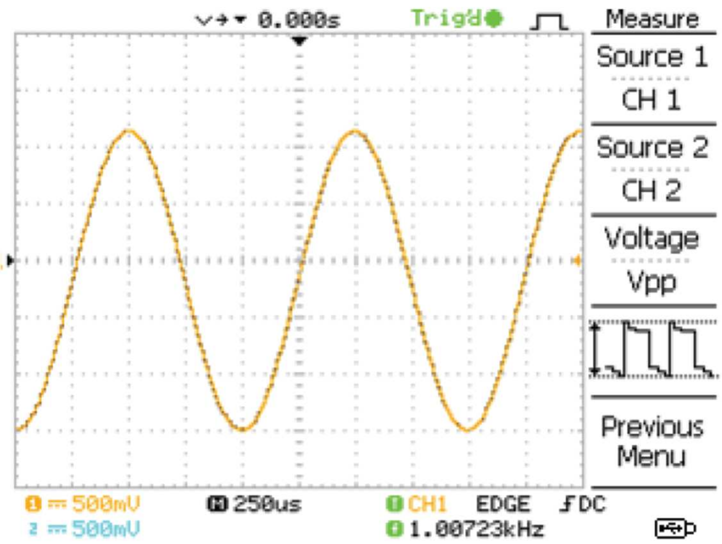
Newer digital oscilloscopes have extra features that can speed your measurements in the Lab. One such feature is the automatic measurement function, which can calculate various attributes of the input signals (such as peak-to-peak voltage, root-mean-square value, period, frequency, etc) without the need to read horizontal or vertical divisions. The results are shown on the right corner of the oscilloscope screen next to the function keys.

**CAUTION**: You still need to know how to work with the oscilloscope's vertical and horizontal divisions since this is a skill that will be tested in the exam.

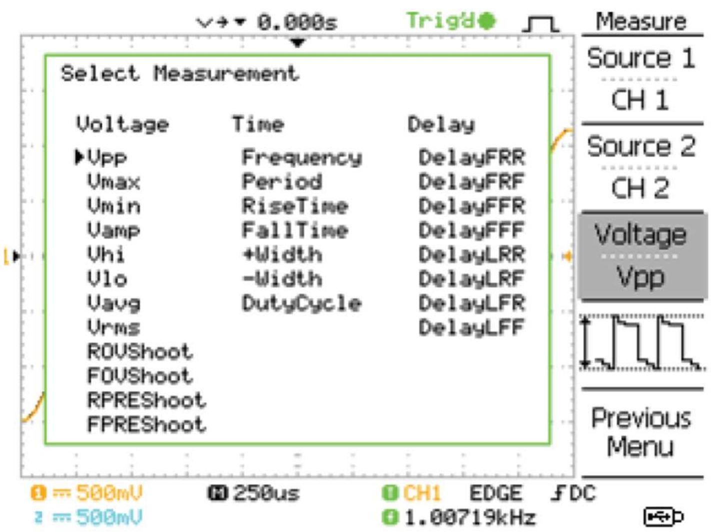
a. On the oscilloscope turn measurement feature ON by pressing the "Measure" key. The default measurements will be shown next to the function keys. The example below shows the oscilloscope measuring peak-to-peak voltage, average (DC) value, frequency, duty cycle, and rise time for both CH1 and CH2 of the oscilloscope.



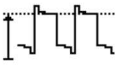
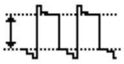
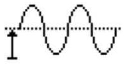
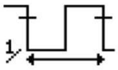
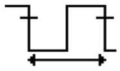
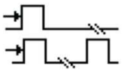
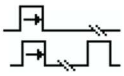

b. To change any of these measurements into something else, click on the function key next to that measurement. The "view/select measurement" menu will be shown (see below) from which you can select the input signals to be measured and the type of measurement for that measurement slot.



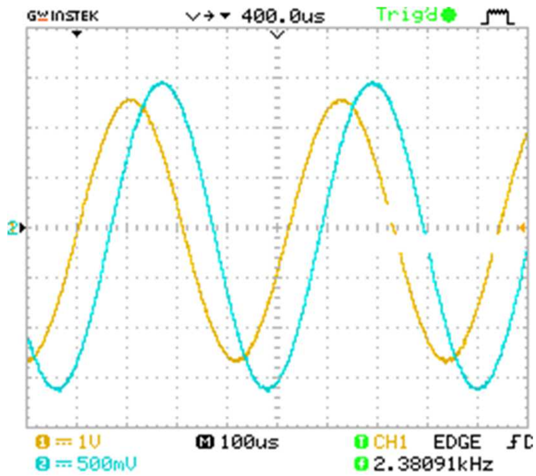

c. Click on the third function key, then use the Variable knob to select a different measurement.



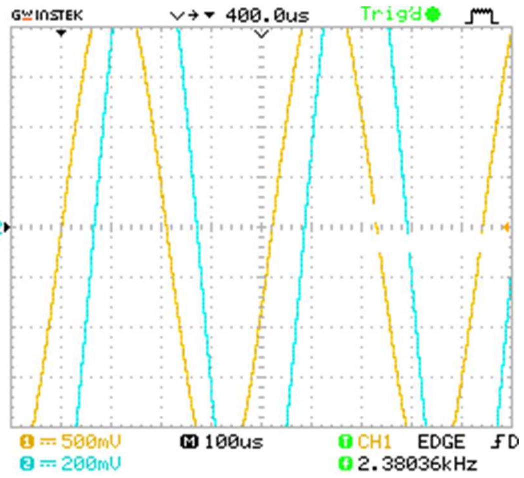
d. Here are some examples of important measurements the oscilloscope can make:

| | | |
|---|---|---|
| Vpp |  | Difference between positive and negative peak voltage (=Vmax − Vmin) |
| Vhi |  | Global high voltage |
| Vamp |  | Difference between global high and global low voltage (=Vhi − Vlo) |
| Vavg |  | Averaged voltage of the first cycle. |
| Vrms |  | RMS (root mean square) voltage. |
| Freq |  | Frequency of the waveform |
| Period |  | Waveform cycle time (=1/Freq). |
| FRR |  | Time between: Source 1 first rising edge and Source 2 first rising edge |
| FFF |  | Time between: Source 1 first falling edge and Source 2 first falling edge |

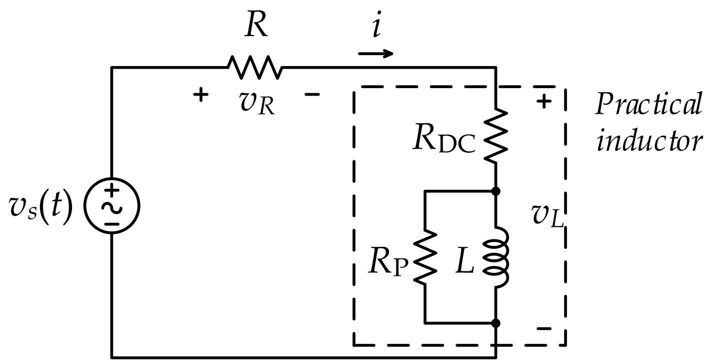
e. To get correct measurements of the signal properties, the signal needs to be contained within the oscilloscope screen limits as in the following example:



f. However, if you are not careful enough and the signal exceeds the oscilloscope screen limits, you might get wrong measurements (or no measurements at all) as in the following example:




10. You might have noticed some discrepancies between theoretical and measured values in Tables 1 and 2 for $\boldsymbol{V_L}$ and $\boldsymbol{V_R}$ values at some frequencies. The reason this happens is that a practical inductor is not exactly an ideal inductor. Rather, it has internal resistances (representing the internal resistance of the long wound wire making up the inductor and another resistance representing low power core losses in the inductor). A more realistic model for a practical inductor is shown below. The following complex model is one of several reasons (including the high cost of inductors) why it is much harder to work with inductors in real life compared to resistors and capacitors.




11. Now, using the values in Tables 1 and 2, evaluate the reactance of the inductor $\boldsymbol{X_L}$ and the total impedance $\boldsymbol{Z}$ of the series $R$ and $L$ components, and record them in Table 3. Remember that the total impedance $\boldsymbol{Z}$ is a complex number, so you need to find both its magnitude and phase.

12. Using the *measured* values in Table 3, plot (**by hand**) the following figures using the graph paper attached at the end of the report: (1) $\boldsymbol{X_L}$ and $|\boldsymbol{Z}|$ on the same plot versus source frequency; (2) $\measuredangle \boldsymbol{Z}$ versus source frequency; (3) $\boldsymbol{V_L}$ and $\boldsymbol{V_R}$ on the same plot versus source frequency.

13. For the above plots, state your conclusions under the plot?

**Table 3**

| AC Source Frequency (Hz) | $X_L = \|V_L\|/\|I\|$ (peak/peak) (kΩ) | | $\|Z\| = \|V_S\|/\|I\|$ (peak/peak) (kΩ) | | $\measuredangle Z = \measuredangle V_S - \measuredangle I$ (degrees) | |
|---|---|---|---|---|---|---|
| | Theory | Meas. | Theory | Meas. | Theory | Meas. |
| 700 | | | | | | |
| 1300 | | | | | | |
| 2600 | | | | | | |
| 5200 | | | | | | |
| 9100 | | | | | | |
| 14400 | | | | | | |
| 22200 | | | | | | |
| 39200 | | | | | | |

14. Using the *measured* values of $\boldsymbol{V_S}$ and $\boldsymbol{I}$ in Tables 1 and 2, evaluate the apparent power, real power, and reactive power generated by the source (function generator) and record them in Table 4 below. Also find the power factor (PF) and state whether it is leading or lagging.

15. Using the values in Table 4, plot (**by hand**) the following figure using the graph paper attached at the end of the report: $\boldsymbol{P}$ and $\boldsymbol{Q}$ on the same plot versus source frequency.

16. For the above plot, state your conclusions under the plot?

17. At what frequency the real power $\boldsymbol{P}$ is maximum? Why?

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

18. At what frequency the magnitude of the reactive power $|\boldsymbol{Q}|$ is maximum? Why?

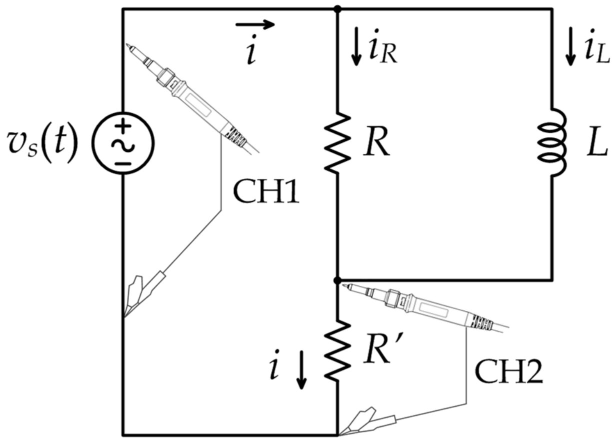
. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

**Table 4**

| AC Source Frequency (Hz) | $\|S\|$ (mVA) | $\measuredangle S$ (degrees) | $P$ (mW) | $Q$ (mVAR) | $PF$ value | $PF$ lead or lag |
|---|---|---|---|---|---|---|
| | Measured | Measured | Measured | Measured | Measured | Measured |
| 700 | | | | | | |
| 1300 | | | | | | |
| 2600 | | | | | | |
| 5200 | | | | | | |
| 9100 | | | | | | |
| 14400 | | | | | | |
| 22200 | | | | | | |
| 39200 | | | | | | |

## PROCEDURE B – AC-EXCITED PARALLEL RL CIRCUIT

1. Construct the circuit shown below. Use $R$ = 510 Ω, $L$ = 10 mH, and $R'$ = 10 Ω. Make sure you use the correct resistor values.




2. Set the function generator to produce a sinusoidal waveform (AC) with frequency of 810 Hz, and *peak voltage* of $V_p = 3$ V.

**CAUTION:** Some older function generators have a defect and produce an AC signal with a slight DC shift (positive or negative). Hence, if you do not see a symmetric sinusoidal signal above and below zero volts, adjust the DC offset knob slightly to force a zero DC offset in the function generator output.

3. Use theoretical analysis to determine the currents $\boldsymbol{I_L}$, $\boldsymbol{I_R}$ and $\boldsymbol{I}$ in the circuit at the different frequencies in Tables 5, 6 and 7. Make sure to evaluate both magnitude and phase for each complex quantity. Record the answers in these tables? The small resistor $R'$ was placed as a convenient way to measure the current $\boldsymbol{I}$ using the oscilloscope. It has a small resistance compared to other resistors in the circuit, which means you can neglect it in theoretical calculations.

4. Now use the oscilloscope to measure the peak value of the voltage $\boldsymbol{V_{R'}}$ and its phase shift compared to $\boldsymbol{V_S}$. If you want, change the horizontal sweep settings of the oscilloscope to make more accurate measurements of the phase. Record the measurements in Table 5?

**CAUTION:** Whenever you change the frequency of the function generator, verify the period of the signal from the oscilloscope to get accurate readings. Also re-check the peak-to-peak voltage as the function generator might change the amplitude when you change the frequency.

5. As you change the frequency of the source, use the oscilloscope to also measure the peak value of the voltage $\boldsymbol{V_S} \approx \boldsymbol{V_R}$ and record its phase shift as 0° compared to $\boldsymbol{V_S}$**.** Record the measurements in Table 5**.**

6. Use Ohm's law on $R'$ to evaluate the current $\boldsymbol{I}$ from $\boldsymbol{V_{R'}}$, and Ohm's law on $R$ to evaluate the current $\boldsymbol{I_R}$ from $\boldsymbol{V_R}$. Record the results in Table 6.

7. In Table 7, use phasor subtraction to find $\boldsymbol{I_L} = \boldsymbol{I} - \boldsymbol{I_R}$ from the measured values in previous tables.

**Table 5**

| AC Source Frequency (Hz) | $V_{R'}$ (peak) (V) | | $\measuredangle V_{R'}$ with $V_s$ (Lag = negative) | | $V_R = V_L \approx V_S$ (peak) (V) | | $\measuredangle V_R$ with $V_s$ (degrees) | |
|---|---|---|---|---|---|---|---|---|
| | Theory | Meas. | Theory | Meas. | Theory | Meas. | Theory | Meas. |
| 810 | | | | | 3 | | 0° | 0° |
| 1620 | | | | | 3 | | 0° | 0° |
| 4100 | | | | | 3 | | 0° | 0° |
| 5700 | | | | | 3 | | 0° | 0° |
| 8100 | | | | | 3 | | 0° | 0° |
| 11400 | | | | | 3 | | 0° | 0° |
| 17900 | | | | | 3 | | 0° | 0° |
| 30000 | | | | | 3 | | 0° | 0° |

**Table 6**

| AC Source Frequency (Hz) | $I$ (peak) (mA) $= V_{R'}/R'$ | | $\measuredangle I$ with $V_s$ (Lag = negative) | | $I_R$ (peak) (mA) $= V_R/R$ | | $\measuredangle I_R$ with $V_s$ (degrees) | |
|---|---|---|---|---|---|---|---|---|
| | Theory | Meas. | Theory | Meas. | Theory | Meas. | Theory | Meas. |
| 810 | | | | | | | 0° | 0° |
| 1620 | | | | | | | 0° | 0° |
| 4100 | | | | | | | 0° | 0° |
| 5700 | | | | | | | 0° | 0° |
| 8100 | | | | | | | 0° | 0° |
| 11400 | | | | | | | 0° | 0° |
| 17900 | | | | | | | 0° | 0° |
| 30000 | | | | | | | 0° | 0° |

**Table 7**

| AC Source Frequency (Hz) | $I_L = I - I_R$ (mA) (magnitude (peak) and phase (degrees)) (phasor subtraction) | |
|---|---|---|
| | Theory | Measured |
| 810 | | |
| 1620 | | |
| 4100 | | |
| 5700 | | |
| 8100 | | |
| 11400 | | |
| 17900 | | |
| 30000 | | |

8. Can we just subtract the magnitudes of $|\boldsymbol{I}| - |\boldsymbol{I_R}|$ to obtain the magnitude $|\boldsymbol{I_L}|$? Why or why not?

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

9. Using the values in Tables 5, 6 and 7, evaluate the susceptance of the inductor $\boldsymbol{B_L}$ and the total admittance $\boldsymbol{Y}$ of the parallel *R* and *L* components, and record them in Table 8. Remember that the total admittance $\boldsymbol{Y}$ is a complex number, so you need to find both its magnitude and phase.

**Table 8**

| **AC Source Frequency (Hz)** | $\boldsymbol{B_L = \lvert I_L\rvert/\lvert V_L\rvert}$ **(peak/peak) (mS)** | | $\boldsymbol{\lvert Y\rvert = \lvert I\rvert/\lvert V_S\rvert}$ **(peak/peak) (mS)** | | $\boldsymbol{\measuredangle Y = \measuredangle I - \measuredangle V_S}$ **(degrees)** | |
|---|---|---|---|---|---|---|
| | **Theory** | **Meas.** | **Theory** | **Meas.** | **Theory** | **Meas.** |
| **810** | | | | | | |
| **1620** | | | | | | |
| **4100** | | | | | | |
| **5700** | | | | | | |
| **8100** | | | | | | |
| **11400** | | | | | | |
| **17900** | | | | | | |
| **30000** | | | | | | |

10. Using the *measured* values in Table 8, plot (**by hand**) the following figures using the graph paper attached at the end of the report: (1) $\boldsymbol{B_L}$ and $|\boldsymbol{Y}|$ on the same plot versus source frequency; (2) $\measuredangle\boldsymbol{Y}$ versus source frequency; (3) $\boldsymbol{I_L}$ and $\boldsymbol{I_R}$ on the same plot versus source frequency.

11. For the above plots, state your conclusions under the plot?

12. Using the *measured* values of $\boldsymbol{V_S} = \boldsymbol{V_R}$ and $\boldsymbol{I}$ in Tables 5 and 6, evaluate the apparent power, real power, and reactive power generated by the source (function generator) and record them in Table 9 below. Also find the power factor (PF) and state whether it is leading or lagging.

13. Using the values in Table 9, plot (**by hand**) the following figure using the graph paper attached at the end of the report: $\boldsymbol{P}$ and $\boldsymbol{Q}$ on the same plot versus source frequency.

14. For the above plot, state your conclusions under the plot?

**Table 9**

| AC Source Frequency (Hz) | $\|S\|$ (mVA) | $\measuredangle S$ (degrees) | $P$ (mW) | $Q$ (mVAR) | $PF$ value | $PF$ lead or lag |
|---|---|---|---|---|---|---|
| | Measured | Measured | Measured | Measured | Measured | Measured |
| 810 | | | | | | |
| 1620 | | | | | | |
| 4100 | | | | | | |
| 5700 | | | | | | |
| 8100 | | | | | | |
| 11400 | | | | | | |
| 17900 | | | | | | |
| 30000 | | | | | | |

15. At what frequency the real power $\boldsymbol{P}$ is maximum? Why?

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16. At what frequency the magnitude of the reactive power $|\boldsymbol{Q}|$ is maximum? Why?

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**** End ****