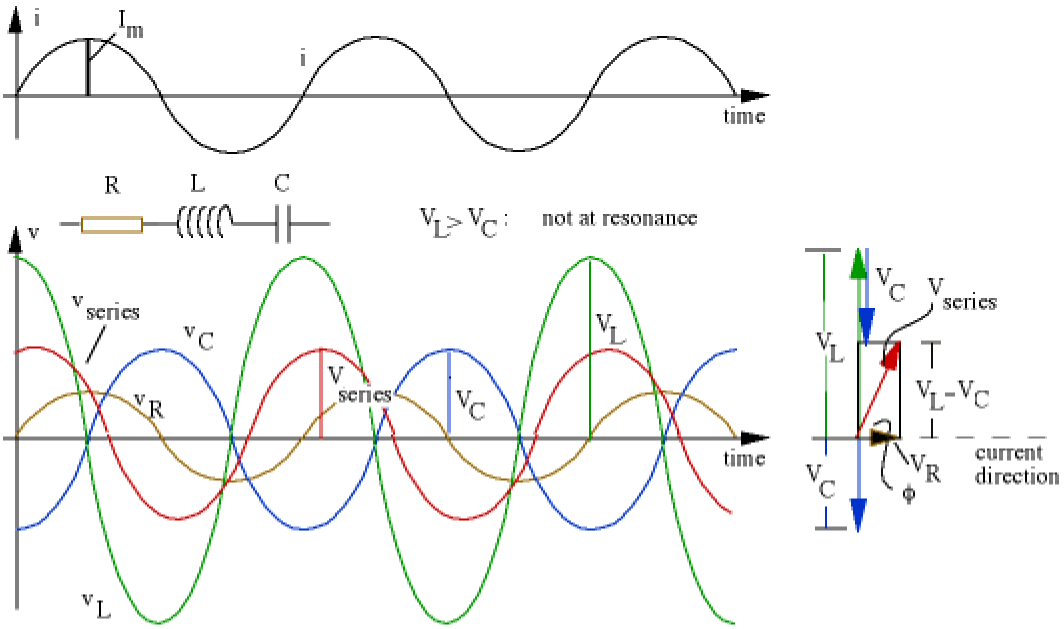
# Explanation of Experiment 6
# Characteristics of series R-L-C Circuits

Now let's put a resistor, capacitor and inductor in series. At any given time, the voltage across the three components in series, $V_{series}(t)$, is the sum of these:

$V_{series}(t) = V_R(t) + V_L(t) + V_C(t)$,

The current i(t) we shall keep sinusoidal, as before. The voltage across the resistor, $V_R(t)$ ,is in phase with the current. That across the inductor, $V_L(t)$, is 90° ahead and that across the capacitor, $V_C(t)$), is 90° behind.

Once again, the time-dependent voltages v(t) add up at any time, but the RMS voltages V do not simply add up. Once again they can be added by phasor representing the three sinusoidal voltages. Again, let's 'freeze' it in time for the purposes of the addition, which we do in the graphic below. Once more, be careful to distinguish v and V.




Look at the phasor diagram: The voltage across the ideal inductor is anti-parallel to that of the capacitor, so the total reactive voltage (the voltage which is 90° ahead of the current) is VL - VC, so Pythagoras now gives us:

$$V_{Series}{}^2 = V_R{}^2 + (V_L - V_c)^2$$

$V^2_{series} = V_R{}^2 + (VL - VC)2$

Now $V_R = IR$, $V_L = IX_L = \omega L$ and $V_C = IX_C = 1/\omega C$. Substituting and taking the common factor I gives:

Where ω=2π*f*

$$V = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = I Z_{series}$$

where $Z_{series}$ is the series impedance: the ratio of the voltage to current in an RLC series circuit. Note that, once again, reactance and resistances add according to Pythagoras' law:

$$Z_{Series}{}^2 = R^2 + X_{total}{}^2$$

$$Z_{Series}{}^2 = R^2 + (X_L - X_c)^2$$

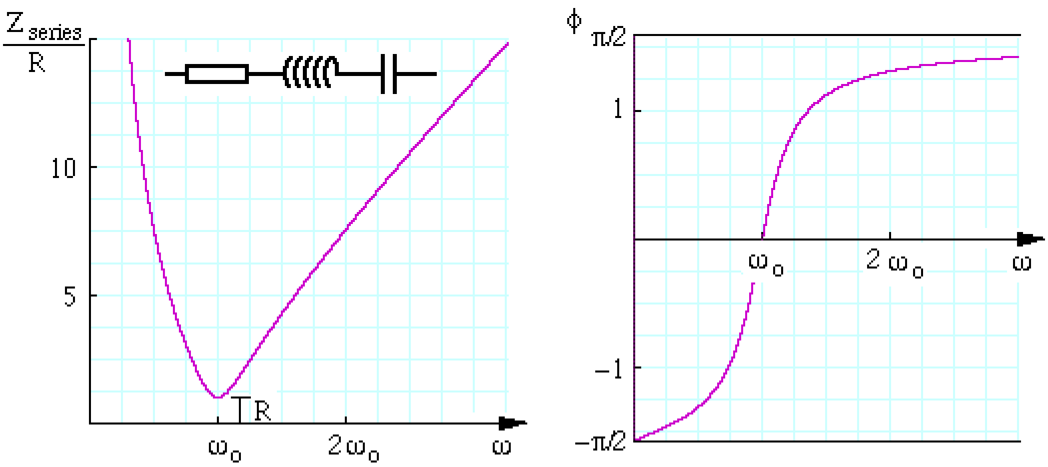
Remember that the inductive and capacitive phasor are 180° out of phase, so their reactance tend to cancel.

Now let's look at the relative phase. The angle by which the voltage leads the current is

$$\theta = \tan^{-1}\left(\frac{(V_L - V_C)}{V_R}\right)$$

Substituting $V_R$ = IR, $V_L$ = $IX_L$ = ωL and $V_C$ = $IX_C$= 1/ωC gives:

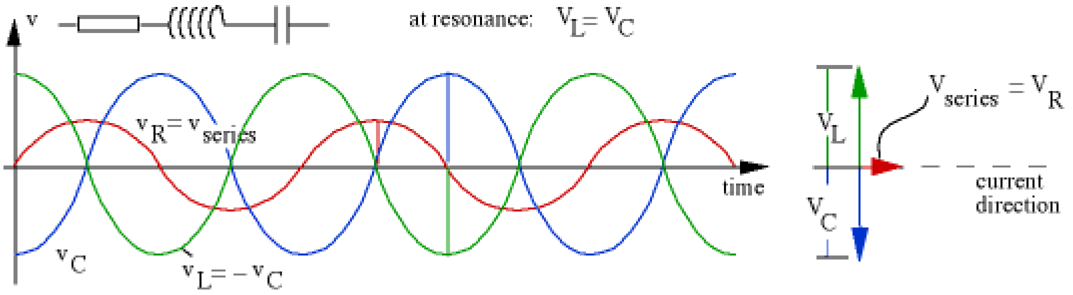
The dependence of $Z_{series}$ and θ on the angular frequency ω is shown in the next figure. The angular frequency ω is given in terms of a particular value $\omega_o$, the resonant frequency ($\omega_o^2$ = 1/LC), which we meet below.




$$Z_{series} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad \phi = \tan^{-1}\frac{\omega L - 1/\omega C}{R}$$

(Setting the inductance term to zero gives back the equations we had above for RC circuits, though note that phase is negative, meaning (as we saw above) that voltage lags the current. Similarly, removing the capacitance terms gives the expressions that apply to RL circuits.)

The next graph shows us the special case where the frequency is such that $V_L$ = $V_C$.




Because $V_L(t)$ and $V_C$ are 180° out of phase, this means that $V_L(t) = - V_C(t)$, so the two reactive voltages cancel out, and the series voltage is just equal to that across the resistor. This case is called series resonance, which is our next topic.

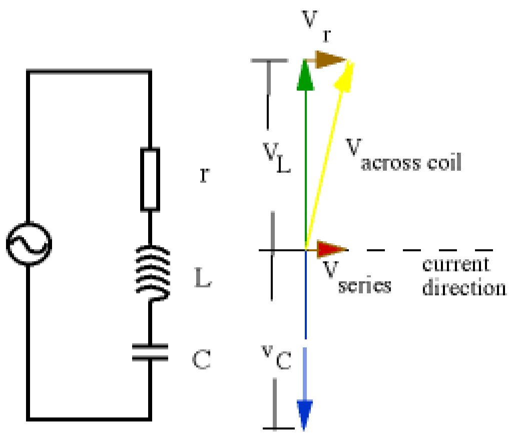
## Resonance:

Note that the expression for the series impedance goes to infinity at high frequency because of the presence of the inductor, which produces a large *emf* if the current varies rapidly. Similarly it is large at very low frequencies because of the capacitor, which has a long time in each half cycle in which to charge up. when the voltages across capacitor and inductor are equal and opposite, ie $V_L(t) = - V_C(t)$ so $V_L(t) = V_C$, so

$\omega L = {}^{1}/_{\omega C}$ so the frequency at which this occurs is

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

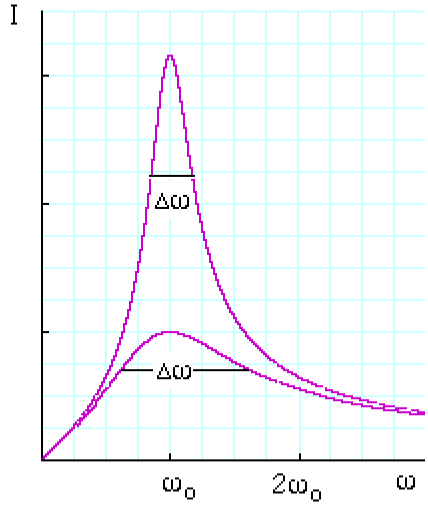



where $\omega_o$ and $f_o$ are the angular and cyclic frequencies of resonance, respectively. At resonance, series impedance is a minimum, so the voltage for a given current is a minimum (or the current for a given voltage is a maximum).

You get a big voltage in the circuit for only a small voltage input from the power source. You are not, of course, getting something for nothing. The energy stored in the large oscillations is gradually supplied by the AC source when you turn on, and it is then exchanged between capacitor and inductor in each cycle. For more details about this phenomenon, and a discussion of the energies involved, go to LC oscillations.

## Bandwidth and Q factor:

At resonance, the voltages across the capacitor and the pure inductance cancel out, so the series impedance takes its minimum value: $Z_o$ = R. Thus, if we keep the voltage constant, the current is a maximum at resonance. The current goes to zero at low frequency, because XC becomes infinite (the capacitor is open circuit for DC). The current also goes to zero at high frequency because XL increases with ω (the inductor opposes rapid changes in the current). The graph shows I(ω) for circuit with a large resistor (lower curve) and for one with a small resistor (upper curve). A circuit with low R, for a given L and C, has a sharp resonance. Increasing the resistance makes the resonance less sharp. The former circuit is more selective: it produces high currents only for a narrow bandwidth, ie a small range of ω or f. The circuit with higher R responds to a wider range of frequencies and so has a larger bandwidth. The bandwidth Δω (indicated by the horizontal bars on the curves) is defined as the difference between the two frequencies ω+ and ω- at which the circuit converts power at half the maximum rate.




Now the electrical power converted to heat in this circuit is I2R, so the maximum power is converted at resonance, ω = $\omega_o$. The circuit converts power at half this rate when the current is $I_o/\sqrt{2}$ The Q value is defined as the ratio

Q = ωo/Δω.