

**University of Jordan
School of Engineering
Electrical Engineering Department**

**EE 219
Electrical Circuits Lab**

**EXPERIMENT 6
INDUCTIVE REACTANCE**

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EXPERIMENT 6

INDUCTIVE REACTANCE

OBJECTIVE

Inductive reactance will be examined in this experiment. In particular, its relationship to the AC source frequency will be investigated, including a plot of inductive reactance versus frequency. In addition, AC power and power factor calculations will be introduced conducted.

DISCUSSION

Impedance, Reactance, Admittance and Susceptance

The AC current-voltage characteristic of an inductor, known as inductive *reactance* X_L , is directly proportional to frequency. Hence, the *impedance* of an inductor is:

$$\mathbf{Z}_L = jX_L = j\omega L$$

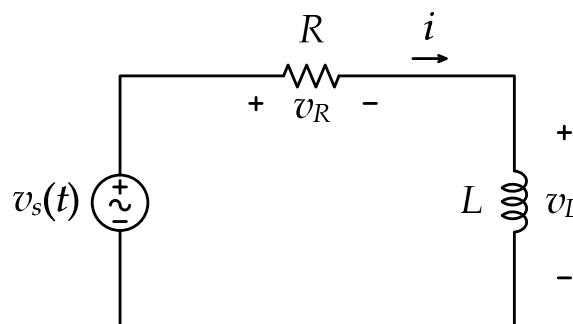
The inverse of the impedance is *admittance* $\mathbf{Y} = 1/\mathbf{Z} = G + jB$, which has the real part G called *conductance*, and the imaginary part B called *susceptance*. All three quantities have units of Siemens (S).

The inductive reactance may be determined experimentally by applying a known AC voltage across the inductor, measuring the resulting current, and dividing the two. This process may be repeated across a range of frequencies in order to obtain a plot of inductive reactance versus frequency.

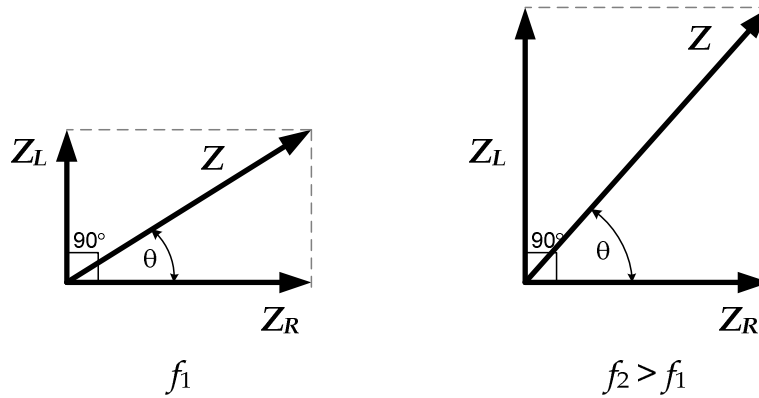
AC-excited series RL circuit

For the series RL circuit shown below, the total impedance seen by the AC source is given by:

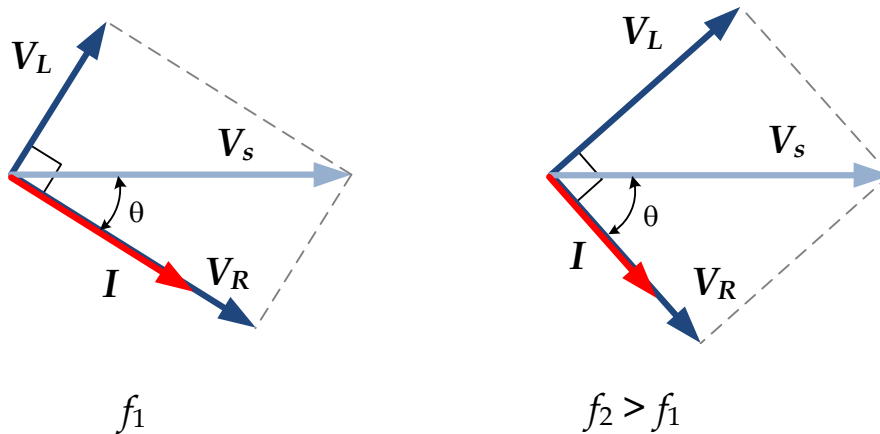
$$\mathbf{Z} = \mathbf{Z}_R + \mathbf{Z}_L = R + j\omega L = \sqrt{R^2 + (\omega L)^2} \angle \tan^{-1}\left(\frac{\omega L}{R}\right)$$



Since the inductor impedance \mathbf{Z}_L changes with the frequency of the source, the total impedance \mathbf{Z} changes with frequency as well, as shown in the following phasor diagram. Notice how the resistor impedance remains constant with frequency.



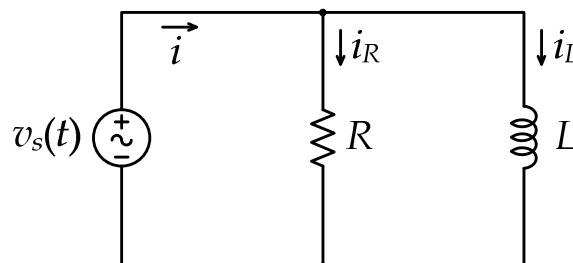
The following phasor diagram shows how the above impedance change affects the inductor voltage V_L and resistor voltage V_R , both of which now change with frequency for a constant V_S due to the voltage divider rule. The current I also changes along with its phase shift compared to the source voltage V_S , but the current remains lagging compared to the source voltage. Notice that complex numbers are added like vectors, not like scalars.



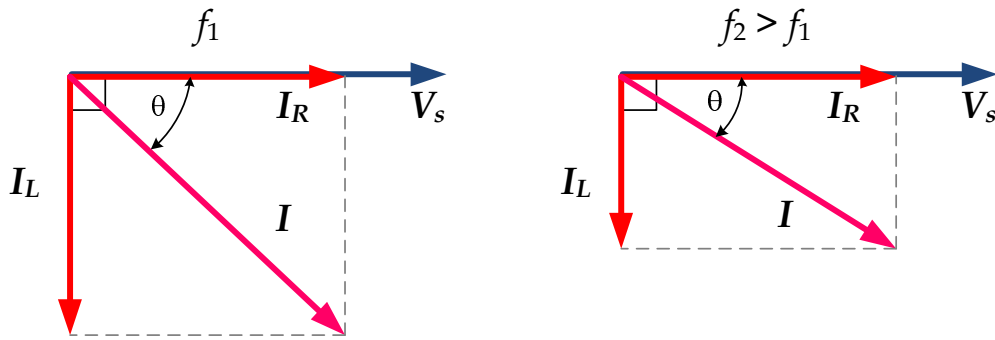
AC-excited parallel RL circuit

For the parallel RL circuit shown below, the total admittance seen by the AC source is given by:

$$Y = Y_R + Y_L = \frac{1}{R} + \frac{1}{j\omega L} = \frac{1}{R} - j\frac{1}{\omega L} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L}\right)^2} \angle -\tan^{-1}\left(\frac{R}{\omega L}\right)$$



The following phasor diagram shows how the inductive admittance change with frequency affects the inductor current I_L , which decreases with increasing frequency for a constant source voltage V_S . The resistance current I_R , however, stays constant, which means that the total current I changes (due to phasor addition) along with its phase shift compared to the source voltage V_S . Of course, the current lags the source voltage due to the inductive load.



Power and Power Factor

The average *complex power* \mathcal{S} (units of VA) is given by the combination of the average *real power* P and the average *reactive power* Q as follows:

$$\mathcal{S} = P + jQ = V_{rms}I_{rms}\angle(\angle V - \angle I) = \frac{1}{2}V_pI_p\angle(\angle V - \angle I)$$

Hence, the real power P (units of W) is given by:

$$P = V_{rms}I_{rms} \cos(\angle V - \angle I) = \frac{1}{2}V_pI_p \cos(\angle V - \angle I)$$

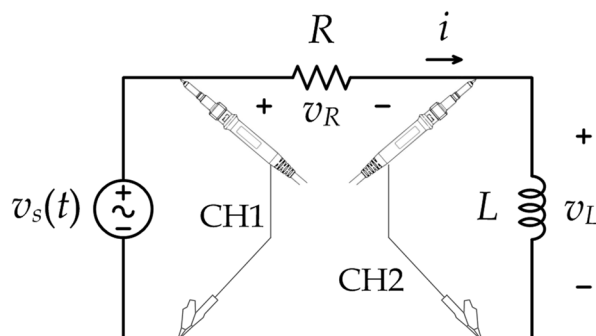
And the reactive power Q (units of VAR) is given by:

$$Q = V_{rms}I_{rms} \sin(\angle V - \angle I) = \frac{1}{2}V_pI_p \sin(\angle V - \angle I)$$

where the $\cos(\angle V - \angle I)$ quantity is known as the *power factor* (PF), which is lagging for inductive loads, and leading for capacitive loads. Finally the *apparent power* is given by $|\mathcal{S}| = V_{rms}I_{rms} = 0.5 \times V_p \times I_p$.

PROCEDURE A - AC-EXCITED SERIES RL CIRCUIT

1. Construct the circuit shown below. Assume that $R = 820 \Omega$, $L = 10 \text{ mH}$.



2. Set the function generator to produce a sinusoidal waveform (AC) with frequency of 700 Hz, and *peak voltage* of $V_p = 2 \text{ V}$.

CAUTION: Some older function generators have a defect and produce an AC signal with a slight DC shift (positive or negative). Hence, if you do not see a symmetric sinusoidal signal above and below zero volts, adjust the DC offset knob slightly to force a zero DC offset in the function generator output.

3. Use theoretical analysis to determine the voltages V_L and V_R and current I in the circuit at the different frequencies in Tables 1 and 2. Make sure to evaluate both magnitude and phase for each complex quantity, and record the expected period T of the signals in milliseconds. Record the answers in the two tables? **Note:** Using MATLAB can quickly give you the theoretical answers if you define a vector of frequencies and then use array arithmetic.

4. Use the oscilloscope to measure the peak values of the voltages V_S and V_L and the phase shift of V_L compared to V_S . Remember that you can change the horizontal sweep setting of the oscilloscope to make more accurate measurements of the phase. Also measure the period T in milliseconds of the source voltage V_S . Record the measurements in Table 1. You can use the “Automatic Measurement” feature of the oscilloscope explained below to speed up your procedure.

CAUTION: Whenever you change the frequency of the function generator, verify the period of the signal from the oscilloscope to get accurate readings. Also re-check the peak-to-peak voltage as the function generator might change the amplitude when you change the frequency.

5. Use the oscilloscope to measure the peak value of the voltage V_R and its phase shift compared to V_S . Also measure the period T in milliseconds of V_R . Record the measurements in Table 2. You can use the “Automatic Measurement” feature of the oscilloscope to speed up your procedure.

CAUTION: Do *not* put CH2 of the oscilloscope across the resistor R while simultaneously measuring V_S using CH1 since this will short circuit the inductor. Rather you can use one of the following tricks: (a) you can swap the locations of L and R in the circuit while keeping the oscilloscope connections unchanged, allowing you to measure V_R , or alternatively (b) you can use the **A – B** mode in the oscilloscope, which subtracts the instantaneous signal on CH1 minus the signal on CH2 and displays the result, which is the instantaneous voltage across the resistor.

6. Which of the above two methods did you decide to use to measure V_R ?

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7. Can we just subtract the magnitudes of $|V_S| - |V_L|$ to obtain the magnitude $|V_R|$? Why or why not?

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8. Now evaluate the *measured* current I and its phase shift compared to V_S by applying Ohm’s law to the resistor using the measured value of V_R (i.e., $I = V_R/R$). Record the answer in Table 2.

9. What is the relationship between the periods T of the two signals V_S and V_R ?

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Table 1

AC Source Frequency (Hz)	V_S (peak) (V)		V_S period T (ms)		V_L (peak) (V)		$\angle V_L$ with V_S (Lead = positive)	
	Theory	Meas.	Theory	Meas.	Theory	Meas.	Theory	Meas.
700	2							
1300	2							
2600	2							
5200	2							
9100	2							
14400	2							
22200	2							
39200	2							

Table 2

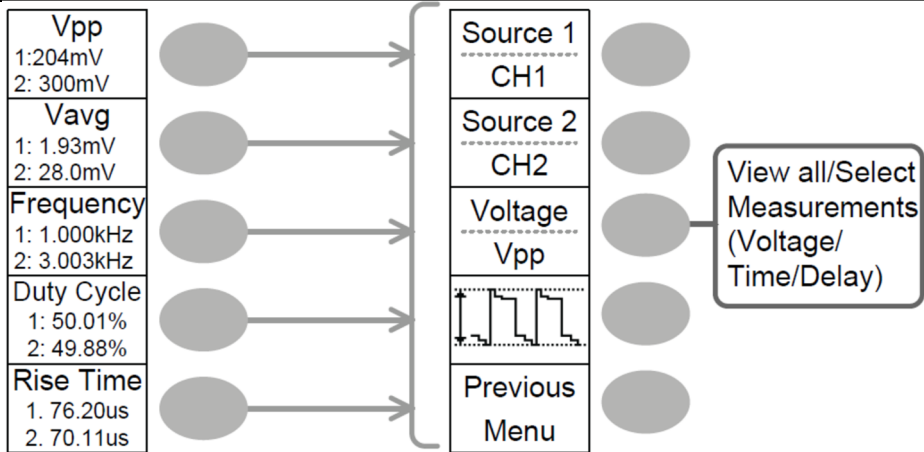
AC Source Frequency (Hz)	V_R (peak) (V)		$\angle V_R$ with V_S (Lag = negative)		V_R period T (ms)		I and $\angle I$ (mA) = V_R/R	
	Theory	Meas.	Theory	Meas.	Theory	Meas.	Theory	Meas.
700								
1300								
2600								
5200								
9100								
14400								
22200								
39200								

OSCILLOSCOPE AUTOMATIC MEASUREMENT

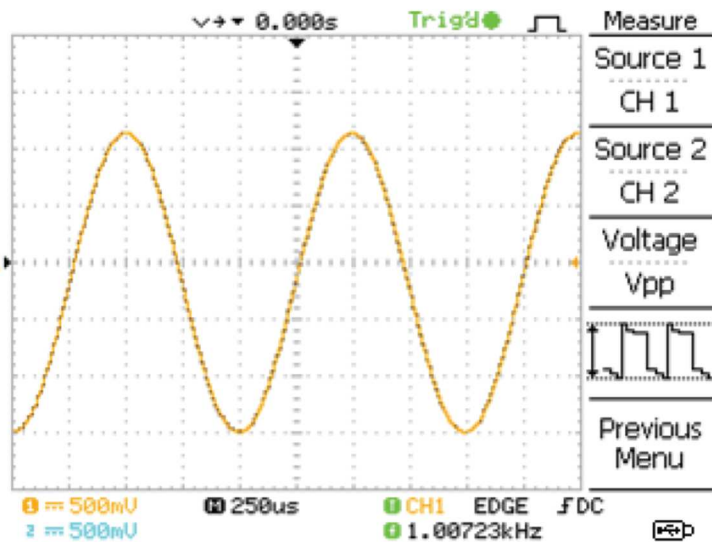
Newer digital oscilloscopes have extra features that can speed your measurements in the Lab. One such feature is the automatic measurement function, which can calculate various attributes of the input signals (such as peak-to-peak voltage, root-mean-square value, period, frequency, etc) without the need to read horizontal or vertical divisions. The results are shown on the right corner of the oscilloscope screen next to the function keys.

CAUTION: You still need to know how to work with the oscilloscope's vertical and horizontal divisions since this is a skill that will be tested in the exam.

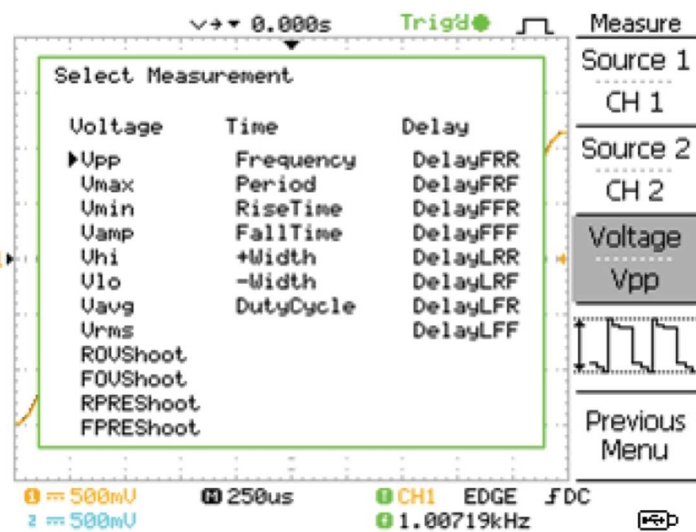
- a. On the oscilloscope turn measurement feature ON by pressing the "Measure" key. The default measurements will be shown next to the function keys. The example below shows the oscilloscope measuring peak-to-peak voltage, average (DC) value, frequency, duty cycle, and rise time for both CH1 and CH2 of the oscilloscope.



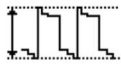

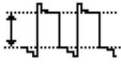
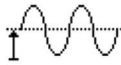


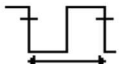


b. To change any of these measurements into something else, click on the function key next to that measurement. The “view/select measurement” menu will be shown (see below) from which you can select the input signals to be measured and the type of measurement for that measurement slot.



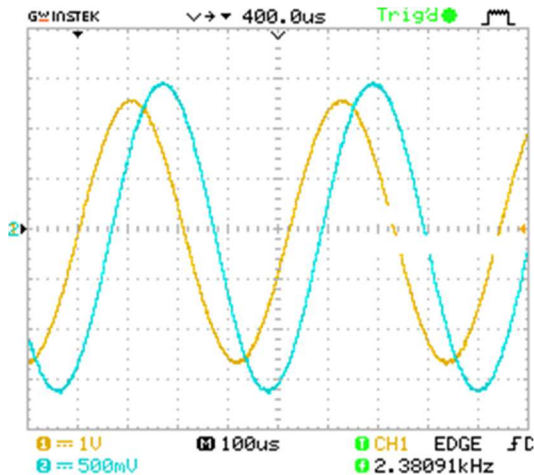
c. Click on the third function key, then use the Variable knob to select a different measurement.



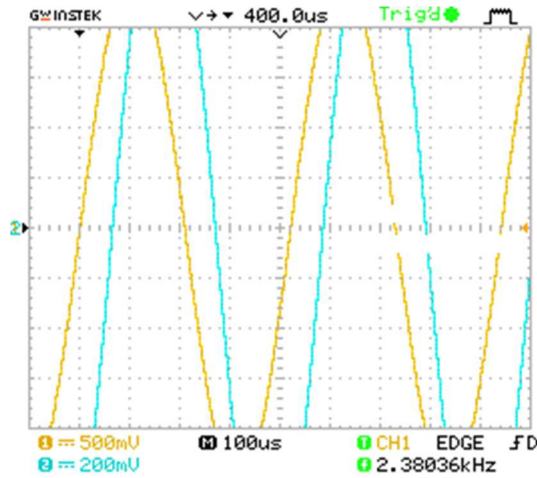
d. Here are some examples of important measurements the oscilloscope can make:

Vpp		Difference between positive and negative peak voltage ($=V_{max} - V_{min}$)
Vhi		Global high voltage
Vamp		Difference between global high and global low voltage ($=V_{hi} - V_{lo}$)
Vavg		Averaged voltage of the first cycle.
Vrms		RMS (root mean square) voltage.
Freq		Frequency of the waveform.
Period		Waveform cycle time ($=1/\text{Freq}$).
FRR		Time between: Source 1 first rising edge and Source 2 first rising edge
FFF		Time between: Source 1 first falling edge and Source 2 first falling edge

e. To get correct measurements of the signal properties, the signal needs to be contained within the oscilloscope screen limits as in the following example:



- f. However, if you are not careful enough and the signal exceeds the oscilloscope screen limits, you might get wrong measurements (or no measurements at all) as in the following example:



10. You might have noticed some discrepancies between theoretical and measured values in Tables 1 and 2 for V_L and V_R values at some frequencies. Why do you think this happened?

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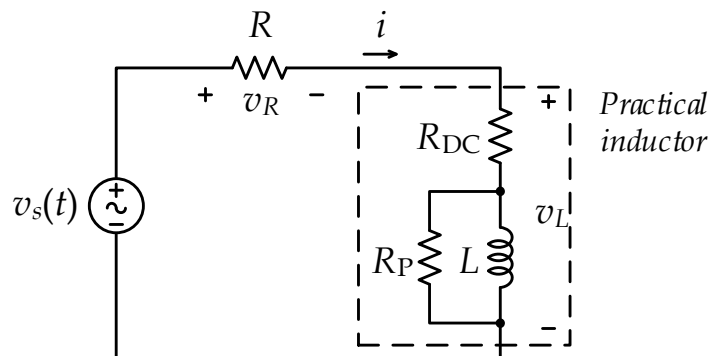
11. Use theoretical analysis to determine the voltages V_L and V_R in the following circuit at frequency **700 Hz**. The circuit shows a more realistic model for the inductor you used (which has an internal resistance for the long wound wire and another resistance representing low power core losses in the inductor). Assume $R_{DC} = 57 \Omega$ and $R_P = 4677 \Omega$. Make sure to evaluate both magnitude and phase for each complex quantity.

$V_L =$ $V_R =$

.....

12. Are the above values closer to your measured values for the same frequency or not?

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13. Use theoretical analysis again for the above circuit to determine the voltages V_L and V_R at frequency **1300 Hz**. Make sure to evaluate both magnitude and phase for each complex quantity.

$V_L =$ $V_R =$

.....

14. Are the above values closer to your measured values for the same frequency or not?

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15. The complex model for practical inductors is one of several reasons why it is much harder to work with inductors in real life compared to resistors and capacitors. Now, using the values in Tables 1 and 2, evaluate the reactance of the inductor X_L and the total impedance Z of the series R and L components, and record them in Table 3. Remember that the total impedance Z is a complex number, so you need to find both its magnitude and phase.

Table 3

AC Source Frequency (Hz)	$X_L = V_L / I $ (peak/peak) (k Ω)		$ Z = V_S / I $ (peak/peak) (k Ω)		$\angle Z = \angle V_S - \angle I$ (degrees)	
	Theory	Meas.	Theory	Meas.	Theory	Meas.
700						
1300						
2600						
5200						
9100						
14400						
22200						
39200						

16. Using the *measured* values in Table 3, plot (**by hand**) the following figures using the graph paper attached at the end of the report: (1) X_L and $|Z|$ on the same plot versus source frequency; (2) $\angle Z$ versus source frequency; (3) V_L and V_R on the same plot versus source frequency.

17. For the above plots, state your conclusions under the plot?

18. Using the *measured* values of V_S and I in Tables 1 and 2, evaluate the apparent power, real power, and reactive power generated by the source (function generator) and record them in Table 4 below. Also find the power factor (PF) and state whether it is leading or lagging.

19. Using the values in Table 4, plot (**by hand**) the following figure using the graph paper attached at the end of the report: P and Q on the same plot versus source frequency.

20. For the above plot, state your conclusions under the plot?

21. At what frequency the real power P is maximum? Why?

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22. At what frequency the magnitude of the reactive power $|Q|$ is maximum? Why?

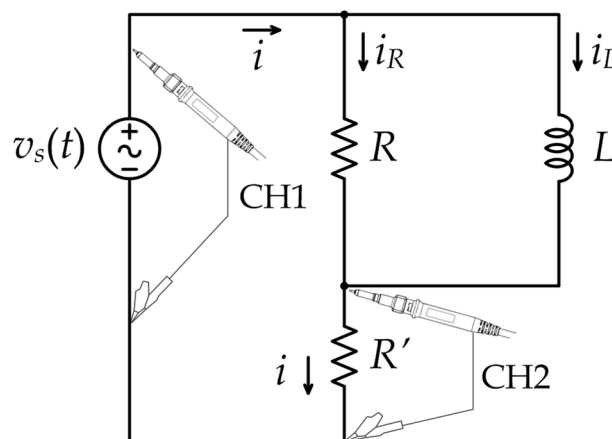
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Table 4

AC Source Frequency (Hz)	$ S $ (mVA)	$\angle S$ (degrees)	P (mW)	Q (mVAR)	PF value	PF lead or lag
	Measured	Measured	Measured	Measured	Measured	Measured
700						
1300						
2600						
5200						
9100						
14400						
22200						
39200						

PROCEDURE B - AC-EXCITED PARALLEL RL CIRCUIT

1. Construct the circuit shown below. Assume that $R = 510 \Omega$, $L = 10 \text{ mH}$, and $R' = 10 \Omega$. Make sure you use the correct resistor values.



2. Set the function generator to produce a sinusoidal waveform (AC) with frequency of 810 Hz, and *peak voltage* of $V_p = 2 \text{ V}$.

CAUTION: Some older function generators have a defect and produce an AC signal with a slight DC shift (positive or negative). Hence, if you do not see a symmetric sinusoidal signal above and below zero volts, adjust the DC offset knob slightly to force a zero DC offset in the function generator output.

3. Use theoretical analysis to determine the currents I_L , I_R and I in the circuit at the different frequencies in Tables 5, 6 and 7. Make sure to evaluate both magnitude and phase for each complex quantity. Record the answers in these tables? The small resistor R' was placed as a convenient way to measure the current I using the oscilloscope. It has a small resistance compared to other resistors in the circuit, which means you can neglect it in theoretical calculations.

4. Now use the oscilloscope to measure the peak value of the voltage $V_{R'}$ and its phase shift compared to V_S . If you want, change the horizontal sweep settings of the oscilloscope to make more accurate measurements of the phase. Record the measurements in Table 5?

CAUTION: Whenever you change the frequency of the function generator, verify the period of the signal from the oscilloscope to get accurate readings. Also re-check the peak-to-peak voltage as the function generator might change the amplitude when you change the frequency.

5. As you change the frequency of the source, use the oscilloscope to also measure the peak value of the voltage $V_S \approx V_R$ and record its phase shift as 0° compared to V_S . Record the measurements in Table 5.

6. Use Ohm's law on R' to evaluate the current I from $V_{R'}$, and Ohm's law on R to evaluate the current I_R from V_R . Record the results in Table 6.

7. In Table 7, use phasor subtraction to find $I_L = I - I_R$ from the measured values in previous tables.

Table 5

AC Source Frequency (Hz)	$V_{R'}$ (peak) (V)		$\angle V_{R'}$ with V_S (Lag = negative)		$V_R = V_L \approx V_S$ (peak) (V)		$\angle V_R$ with V_S (degrees)	
	Theory	Meas.	Theory	Meas.	Theory	Meas.	Theory	Meas.
810					2		0°	0°
1620					2		0°	0°
4100					2		0°	0°
5700					2		0°	0°
8100					2		0°	0°
11400					2		0°	0°
17900					2		0°	0°
30000					2		0°	0°

Table 6

AC Source Frequency (Hz)	I (peak) (mA) = $V_{R'}/R'$		$\angle I$ with V_S (Lag = negative)		I_R (peak) (mA) = V_R/R		$\angle I_R$ with V_S (degrees)	
	Theory	Meas.	Theory	Meas.	Theory	Meas.	Theory	Meas.
810							0°	0°
1620							0°	0°
4100							0°	0°
5700							0°	0°
8100							0°	0°
11400							0°	0°
17900							0°	0°
30000							0°	0°

Table 7

AC Source Frequency (Hz)	$I_L = I - I_R$ (mA) (magnitude (peak) and phase (degrees)) (phasor subtraction)	
	Theory	Measured
810		
1620		
4100		
5700		
8100		
11400		
17900		
30000		

8. Can we just subtract the magnitudes of $|I| - |I_R|$ to obtain the magnitude $|I_L|$? Why or why not?

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9. Using the values in Tables 5, 6 and 7, evaluate the susceptance of the inductor B_L and the total admittance Y of the parallel R and L components, and record them in Table 8. Remember that the total admittance Y is a complex number, so you need to find both its magnitude and phase.

Table 8

AC Source Frequency (Hz)	$B_L = I_L / V_L $ (peak/peak) (mS)		$ Y = I / V_S $ (peak/peak) (mS)		$\angle Y = \angle I - \angle V_S$ (degrees)	
	Theory	Meas.	Theory	Meas.	Theory	Meas.
810						
1620						
4100						
5700						
8100						
11400						
17900						
30000						

10. Using the *measured* values in Table 8, plot (by hand) the following figures using the graph paper attached at the end of the report: (1) B_L and $|Y|$ on the same plot versus source frequency; (2) $\angle Y$ versus source frequency; (3) I_L and I_R on the same plot versus source frequency.

11. For the above plots, state your conclusions under the plot?

12. Using the *measured* values of $V_S = V_R$ and I in Tables 5 and 6, evaluate the apparent power, real power, and reactive power generated by the source (function generator) and record them in Table 9 below. Also find the power factor (PF) and state whether it is leading or lagging.

13. Using the values in Table 9, plot (**by hand**) the following figure using the graph paper attached at the end of the report: P and Q on the same plot versus source frequency.

14. For the above plot, state your conclusions under the plot?

Table 9

AC Source Frequency (Hz)	$ S $ (mVA)	$\angle S$ (degrees)	P (mW)	Q (mVAR)	PF value	PF lead or lag
	Measured	Measured	Measured	Measured	Measured	Measured
810						
1620						
4100						
5700						
8100						
11400						
17900						
30000						

15. At what frequency the real power P is maximum? Why?

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16. At what frequency the magnitude of the reactive power $|Q|$ is maximum? Why?

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**** End ****