Electrical Circuits Lab. 0903219

Series RLC Resonance Circuit

- Series RLC Circuit Resonance Frequency *fr*:

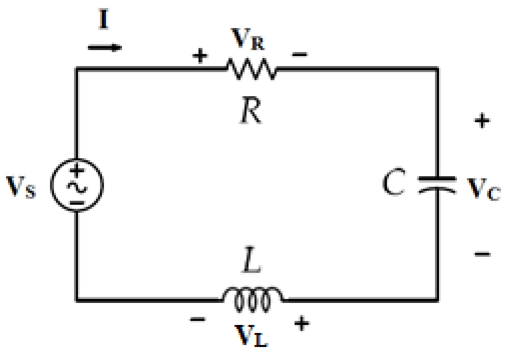


Figure (1) Series RLC circuit

* The definition of the resonance frequency ***fr*** is that it is the operating frequency that makes an **RLC** circuit a resistive circuit which means the imaginary part of the total impedance **Z** becomes zero.

* Depending on the above, we can find a formula for ***fr*** by following the steps shown below:

$$\boldsymbol{When\ f = fr\ in\ a\ series\ RLC\ circuit,}$$

$$Im\{Z\} = 0$$

$$Im\left\{R + j\omega r L + \frac{1}{j\omega r C}\right\} = 0$$

$$j\omega r L + \frac{1}{j\omega r C} = 0$$

$$\rightarrow\ j\omega r L = \frac{-1}{j\omega r C}\ \rightarrow\ \omega r^2 = \frac{1}{LC}$$

$$\rightarrow\ \omega r = \frac{1}{\sqrt{LC}}\ \rightarrow\ fr = \frac{1}{2\pi\sqrt{LC}}.$$

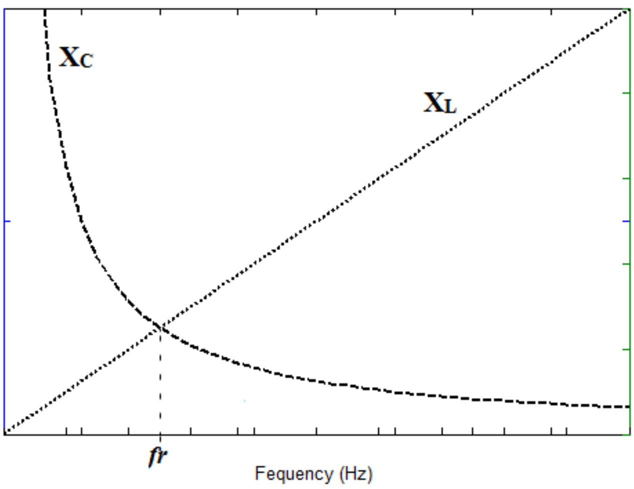


Figure (2) Frequency Response Curves for X_C and X_L reactance.

* Figure (2) shows important plot of how capacitor impedance $\mathbf{X_C}$ and inductor impedance $\mathbf{X_L}$ change with frequency and the place of ***fr*** on the plot (in this case when $\mathbf{X_C}$ equal $\mathbf{X_L}$).

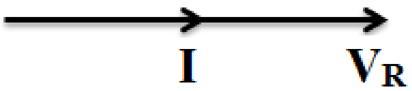
- Simple steps to draw phasor diagram of a series RLC circuit without memorizing! and important conclusions:

* Start with the quantity (voltage or current) that is common for resistor **R**, capacitor **C**, and inductor **L**, which is here the source current **I** (because it passes through all of them without being divided).

Step1

* Now, we know that **I** and resistor voltage $\mathbf{V_R}$ are in phase or have the same phase angle (also in time domain we see that there zero crossings are the same on the time axis) and $\mathbf{V_R}$ is greater than **I** in magnitude.

Step2



* Since **I** equal capacitor current $\mathbf{I_C}$ and equal inductor current $\mathbf{I_L}$, and we know that $\mathbf{I_C}$ leads capacitor voltage $\mathbf{V_C}$ by 90 degrees and $\mathbf{I_L}$ lags inductor voltage $\mathbf{V_L}$ by 90 degrees, both $\mathbf{V_L}$ and $\mathbf{V_C}$ will be on the imaginary axis, and the phasor diagram of a series RLC circuit will have three cases depending on the source operating frequency ***f*** :

a- **Case 1: *f* = *fr***

As mentioned before when ***f* = *fr*** $\underline{\mathbf{X_L = X_C}}$ so $\underline{\mathbf{V_L = V_C}}$ and they are equal in magnitude and out of phase so $\mathbf{V_C}$ and $\mathbf{V_L}$ will cancel each other's effect and the circuit becomes a **<u>resistive circuit</u>** and the phase shift **<u>ϴ equal zero</u>** (remember that ϴ = $\angle \mathbf{I}$ = - $\angle \mathbf{Z}$), the value of **<u>current I is maximum and equals $\mathbf{V_S/R}$</u>** and impedance **<u>Z is minimum and equal R</u>**.

Figure (3) Series RLC Circuit Pharos Diagram when *f* = *fr*

b- **Case 2: *f* < *fr***

Referring to Figure (2) notice that when ***f* < *fr*** $\underline{\mathbf{X_C < X_L}}$ so $\underline{\mathbf{V_C > V_L}}$ and the circuit becomes a **<u>capacitive circuit</u>**, which means that **<u>I leads $\mathbf{V_S}$ and ϴ is a positive angle (with respect to $\mathbf{V_S}$)</u>**.

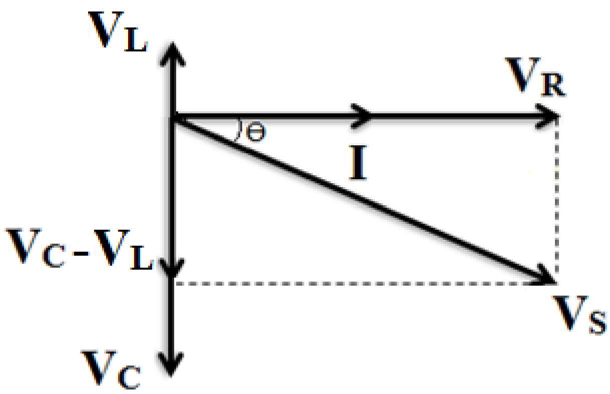


Figure (4) Series RLC Circuit Phasor Diagram when *f* < *fr*

From its phasor diagram in figure (4) we can conclude the following:

1- $|V_S| = \sqrt{(|V_C| - |V_L|)^2 + |V_R|^2}$

2- $\Theta = \tan^{-1} \frac{|V_C| - |V_L| \, (imaginary\ part\ of\ V_S)}{V_R \, (real\ part\ of\ V_S)}$

and remember that ϴ = $\angle \mathbf{I} = -\angle \mathbf{Z} = -\tan^{-1} \frac{X_C - X_L \, \text{(imaginary part of Z)}}{\text{R (real part of Z)}}$

3- $|\mathbf{V_C}|$ and $|\mathbf{V_L}|$ can exceed the source voltage but $|\mathbf{V_C}| - |\mathbf{V_L}|$ and $|\mathbf{V_R}|$ cannot.

c- **Case 3: *f* > *fr***

Referring to Figure (2) notice that when ***f* > *fr*** **X_L>X_C** so **V_L>V_C** and the circuit becomes an **inductive circuit**, which means that **I lags V_S and Θ is a negative angle (with respect to V_S)**.

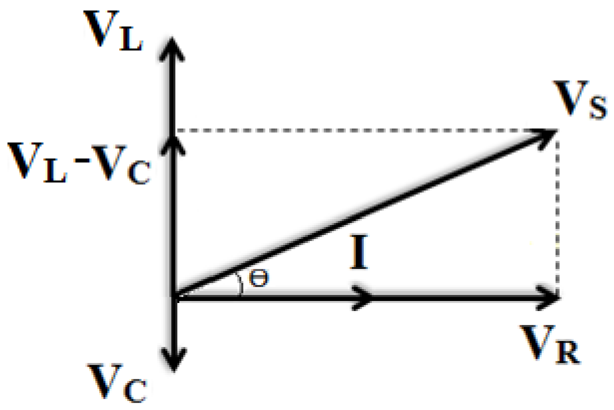


Figure (5) Series RLC Circuit Phasor Diagram when *f* >*fr*

From its phasor diagram in figure (5) we can conclude the following:

1- $|V_S| = \sqrt{(|V_L| - |V_C|)^2 + |V_R|^2}$

2- $\Theta = \tan^{-1}\frac{|V_L|-|V_C|\ (imaginary\ part\ of\ V_S)}{V_R\ (real\ part\ of\ V_S)}$

and remember that $\Theta = \angle I = -\angle Z = -\tan^{-1}\frac{X_L - X_C\ (\text{imaginary part of Z})}{\text{R (real part of Z)}}$

3- |V_C| and |V_L| can exceed the source voltage but **|V_L| - |V_C|** and **|V_R|** cannot.

- How the circuit quantities change with frequency:

* Figure (2) and the circuit phasor diagram helps in finding the circuit quantities change with voltage source frequency ***f*** changing.

* As shown in figure (2), at low frequency ***f*** the difference between **X_C** and **X_L** is huge but with ***f*** increasing this difference starts to decrease so **Z** will decrease until ***f*** reaches ***fr*** where **Z** becomes minimum, after ***f*** exceeds ***fr***, the difference between **X_C** and **X_L** increaeses with frequency increasing so **Z** will increase. In a concise way, the total impedance **Z** will decrease before ***f*** reach ***fr*** then increase when ***f*** exceeds ***fr*** and it's value is minimum at resonance frequency and equals **R** as shown in figure (6).

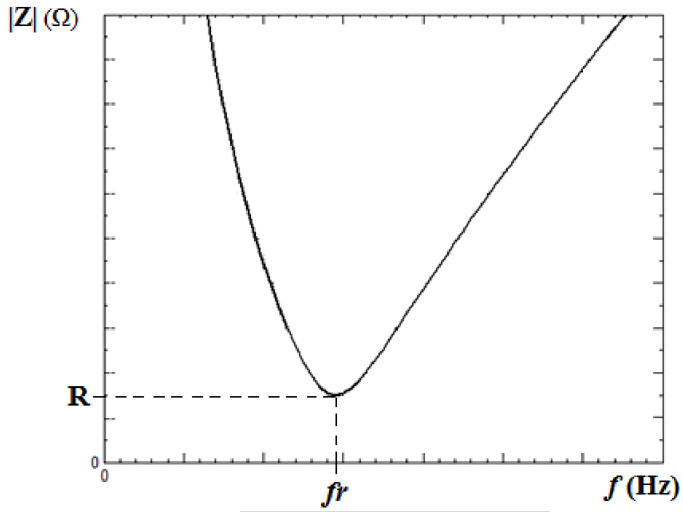


Figure (6) |Z| vs. *f*

* **Θ** ranges from -90° to 90° (-90° < **Θ** < 90°). And since $|\Theta| = \tan^{-1}\frac{|X_L - X_C|}{R}$ and the $\tan^{-1}$ function is increasing on the interval from -90° to 90°, the phase shift **Θ** (or the current angle $\angle$**I**) will decrease before ***f*** reach ***fr*** then increase when ***f*** exceeds ***fr*** and it's value is minimum at resonance frequency and equals **zero** as shown in figure (7).

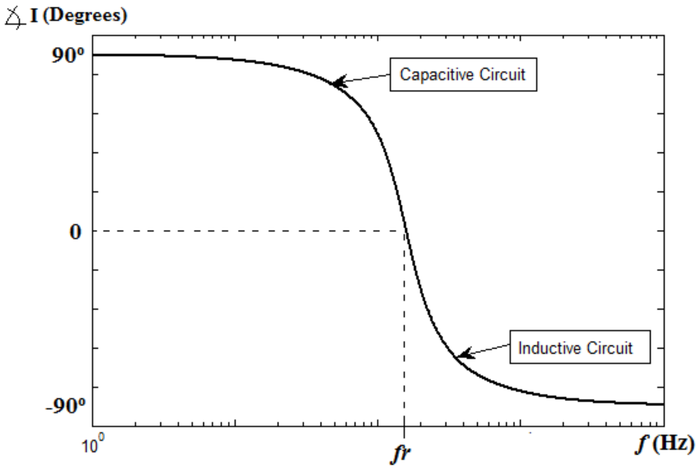


Figure (7) ϴ vs. *f*

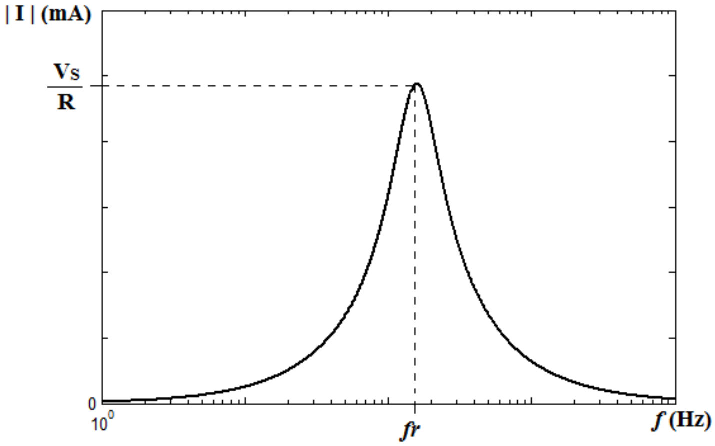


Figure (8) | I | vs. *f*

* Because **I** is inversely proportional to **Z**, the total current **I** will increase before ***f*** reaches ***fr*** then decrease when ***f*** exceeds ***fr*** and it's value is maximum at resonance frequency ***fr*** and equals $\mathbf{V_S/R}$ as shown in figure (8).

* Figure (9) shows $\mathbf{V_C}$,$\mathbf{V_L}$ and $\mathbf{V_R}$ frequency response curves.

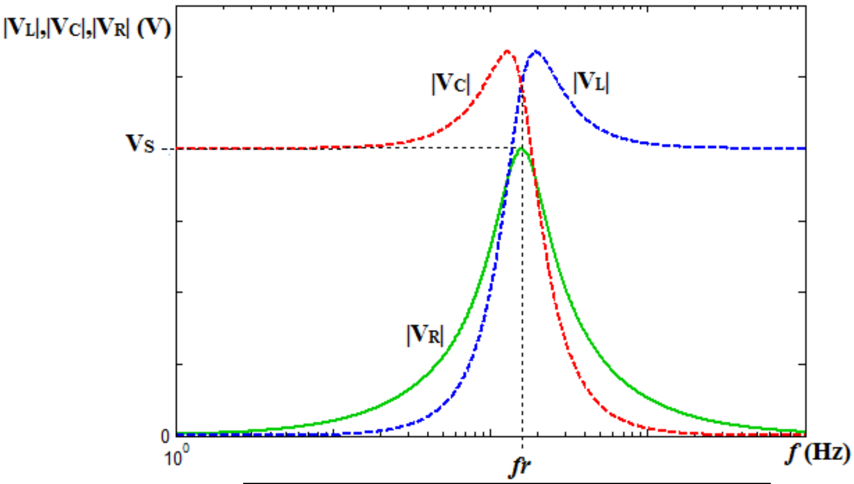


Figure (9) $|\mathbf{V_C}|$,$|\mathbf{V_L}|$ and $|\mathbf{V_R}|$ **vs. *f***

- Figure (10) below shows a time domain representation for all the vectors shown on the phasor diagram for the case *f* < *fr*:

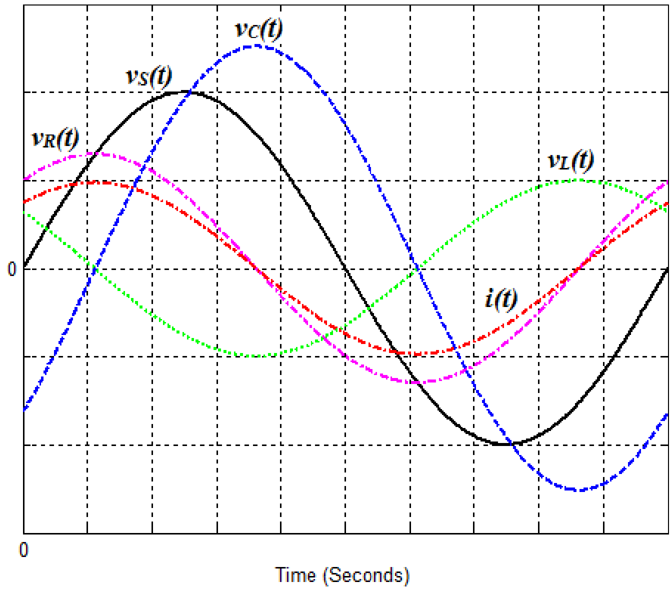


Figure (10) Series RLC Circuit Time Domain Representation

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