Lecture 15: Frequency and Phase Modulation (FM and PM)

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EE421: Communications I: Lecture 15. For more information read Chapter 5 in your textbook or visit http://wikipedia.org/.

Just like *earlier*…

- Time domain expression.
- Time domain *sketch*.
- Average power of modulated signal.
- Frequency domain representation.
- *Bandwidth* of modulated signal.
- Signal-to-Noise Ratio and Quality.
- Practical Applications.
- Modulators and Demodulators (hardware).

Angle Modulation (FM and PM)

$$\varphi_{unmodulated}(t) = A\cos(\omega_c t + \theta_0)$$

$$\varphi_{FM\ or\ PM}(t) = A\cos\theta(t)$$

$$\omega_i(t) \triangleq \frac{d\theta(t)}{dt}$$

$$\theta_i(t) \triangleq \theta(t) - \omega_c t$$

- $\theta(t)$ is **generalized angle** of the modulated signal.
- $\omega_i(t)$ is **instantaneous frequency** of modulated signal.
- $\theta_i(t)$ is **instantaneous phase** of modulated signal.

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Frequency Modulation (FM)

- The *instantaneous frequency* of the modulated signal changes in proportion to the message.

$$\omega_{i_{FM}}(t) = \omega_c + k_f m(t)$$

$$\theta_{FM}(t) = \omega_c t + k_f \int_{-\infty}^{t} m(t)dt$$

$$\varphi_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(t)dt\right)$$

$$\theta_{i_{FM}}(t) = k_f \int_{-\infty}^{t} m(t)dt$$

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Phase Modulation (PM)

- The *instantaneous phase* of the modulated signal changes in proportion to the message.

$$\theta_{i_{PM}}(t) = k_p m(t)$$

$$\theta_{PM}(t) = \omega_c t + k_p m(t)$$

$$\varphi_{PM}(t) = A\cos\left(\omega_c t + k_p m(t)\right)$$

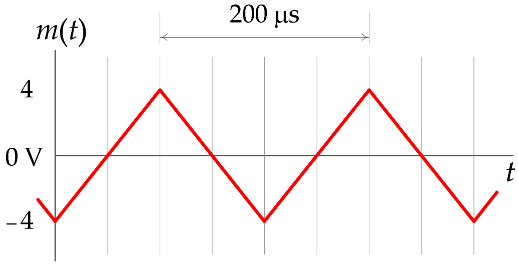
$$\omega_{i_{PM}}(t) = \omega_c + k_P \frac{dm(t)}{dt} = \omega_c + k_P m'(t)$$

FM and PM Equivalence

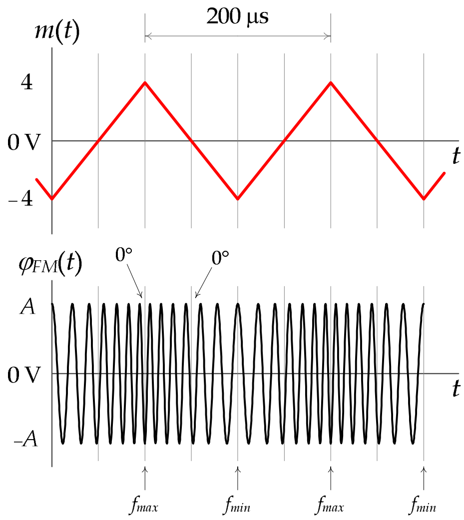
- FM
  - Constant amplitude A
  - Constant carrier frequency ω_c
  - Variable *instantaneous* frequency $\omega_i \propto m(t)$
  - Variable *instantaneous* phase $\theta_i \propto \int m(t)dt$
- PM
  - Constant amplitude A
  - Constant carrier frequency ω_c
  - Variable *instantaneous* frequency $\omega_i \propto m'(t)$
  - Variable *instantaneous* phase $\theta_i \propto m(t)$

Example 1

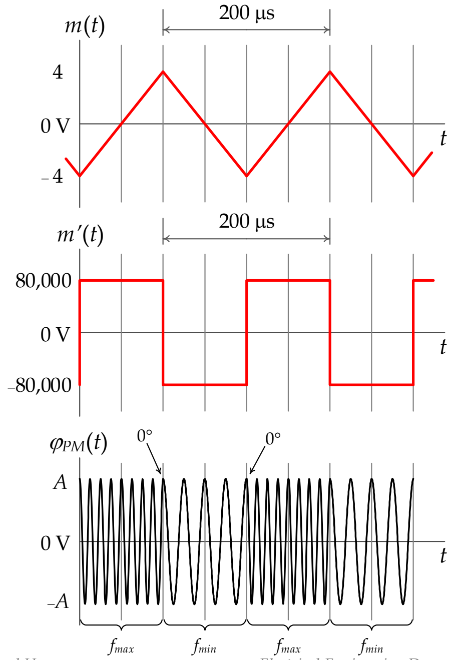
- For the following message signal $m(t)$ and a 100 MHz carrier:
  a) Sketch the **FM** modulated signal. Use $k_f = 2\pi\times10^5$ rad/s/V.
  b) Sketch the **PM** modulated signal. Use $k_p = 5\pi$ rad/V.
  c) Find Δf for both modulated signals.



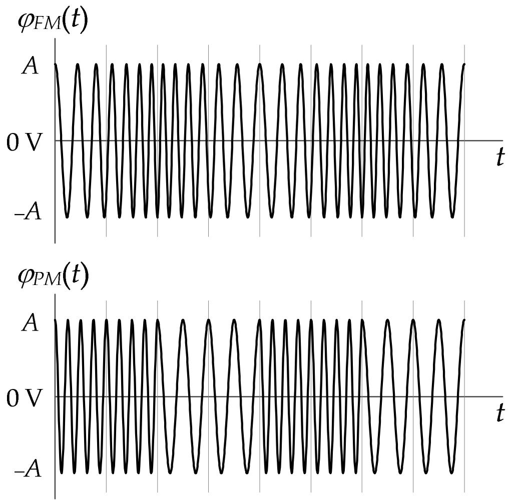
Solution: FM



PM


m(t)
200 μs
4
0 V
–4
t
m′(t)
200 μs
80,000
0 V
–80,000
t
φPM(t)
0°
0°
A
0 V
–A
t
fmax
fmin
fmax
fmin

FM vs. PM


φFM(t)
A
0 V
–A
t
φPM(t)
A
0 V
–A
t

Peak Frequency Deviation

- For **FM**:

$$\Delta f \triangleq \frac{f_{max} - f_{min}}{2} = \frac{k_f}{2\pi} \times \frac{m(t)_{max} - m(t)_{min}}{2}$$

$$\Delta f = \frac{k_f}{4\pi} \times m(t)_{pk-pk} \quad [Hz]$$

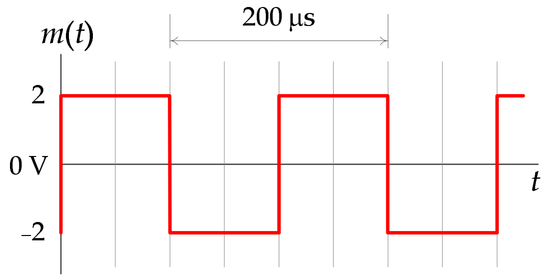
- For **PM**:

$$\Delta f \triangleq \frac{f_{max} - f_{min}}{2} = \frac{k_p}{2\pi} \times \frac{m'(t)_{max} - m'(t)_{min}}{2}$$

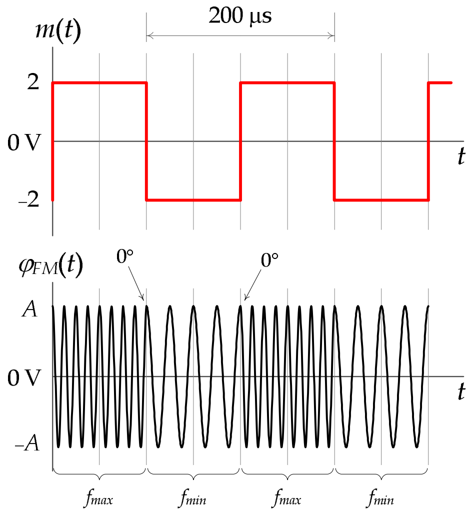
$$\Delta f = \frac{k_p}{4\pi} \times m'(t)_{pk-pk} \quad [Hz]$$

Example 2

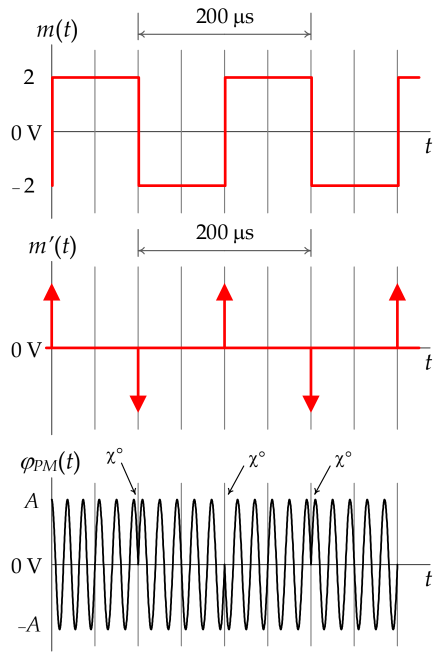
- For the following message signal $m(t)$ and a 100 MHz carrier:
  a) Sketch the **FM** modulated signal. Use $k_f = 2\pi\times10^5$ rad/s/V.
  b) Sketch the **PM** modulated signal. Use $k_p = \pi/4$ rad/V.
  c) Find Δf for both modulated signals.



Solution: FM or FSK

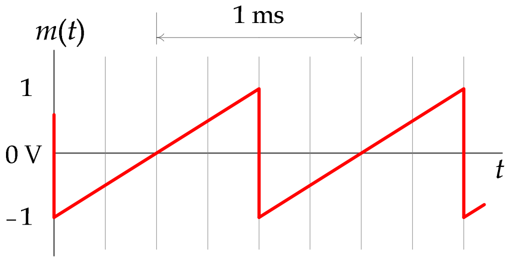


PM or BPSK

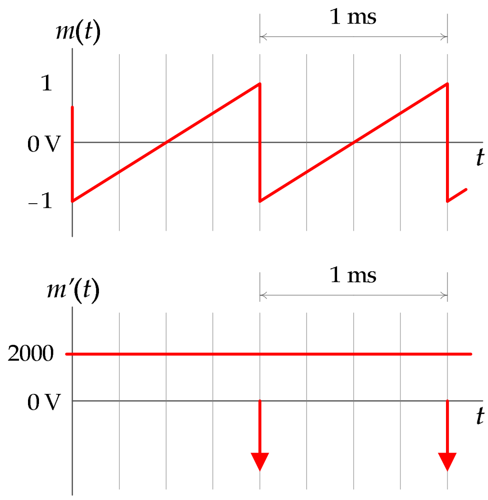


Homework: P.5.1-2

- For the following message signal $m(t)$ and a 200 MHz carrier:
  a) Sketch the **FM** modulated signal. Use $k_f = 2000\pi$ rad/s/V.
  b) Sketch the **PM** modulated signal. Use $k_p = \pi/2$ rad/V.
  c) Try other k_f and k_p values. What is the effect?
  d) Find Δf for both modulated signals.



Hint: For PM



Rules of Thumb

- Smooth change in instantaneous frequency *always* means smooth change in instantaneous phase.
- Sudden change in instantaneous frequency (i.e., unit step change) *does not* mean a sudden change in phase, i.e., it means 0° sudden phase shift.
- *Impulse* change in instantaneous frequency (i.e., infinity frequency) *might* cause a sudden change in phase. To determine the sudden phase shift (or lack thereof) see $k_p m(t)$ for PM or $k_f \int m(t)dt$ for FM.

FM and PM Average Power

$$\phi_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(t)dt\right)$$

$$\phi_{PM}(t) = A\cos\left(\omega_c t + k_p m(t)\right)$$

$$\overline{\varphi_{FM}^2(t)} = \frac{A^2}{2}$$

$$\overline{\varphi_{PM}^2(t)} = \frac{A^2}{2}$$

FM and PM Bandwidth

- Mathematically speaking:
  - $B_{FM} = \infty$
  - $B_{PM} = \infty$
- Practically speaking, use **Carson's Rule**:
  - $B_{FM} \approx 2\Delta f + 2B = 2B(\beta+1)$
  - $B_{PM} \approx 2\Delta f + 2B = 2B(\beta+1)$
- FM Modulation Index:
  - $\beta = \Delta f / B$
  - *Narrow-Band FM* (NBFM) has $\beta \ll 1$ or $\Delta f \ll B$
  - *Wide-Band FM* (WBFM) has $\beta \gg 1$ or $\Delta f \gg B$
  - FM radio uses WBFM with $\beta = 5$

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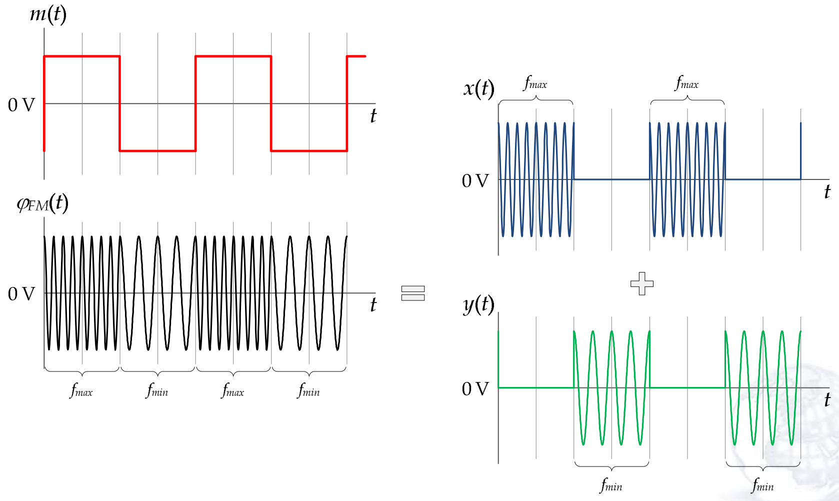
FM Bandwidth: *Semi-proof*



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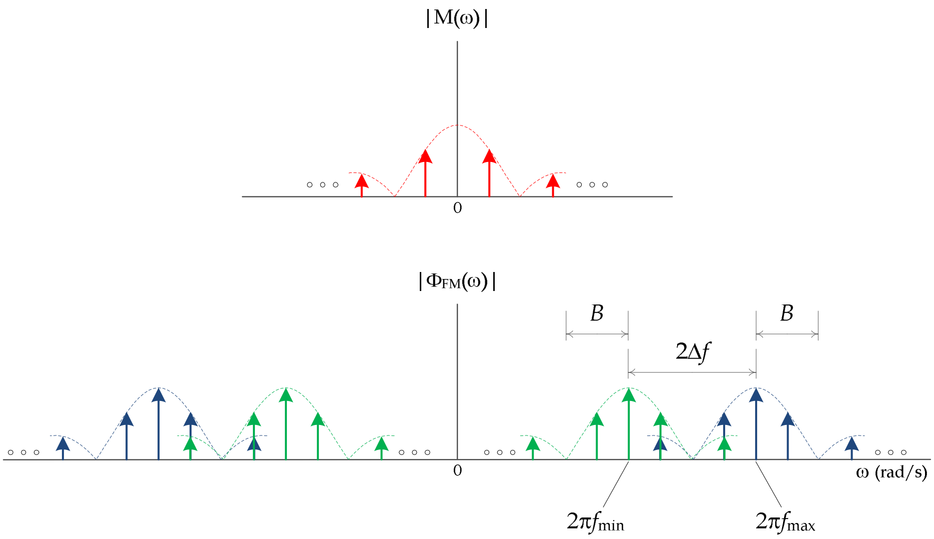
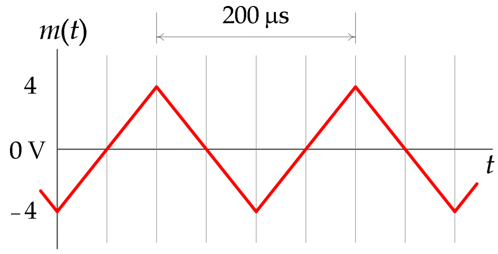


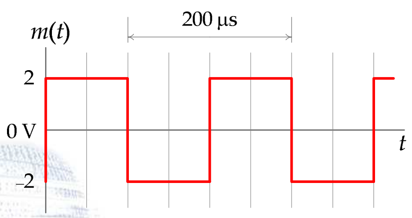
Bandwidth: Example 1

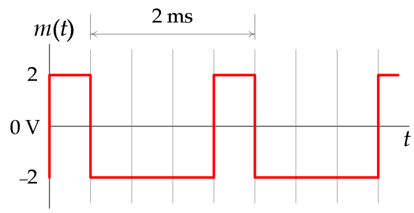
- Estimate the bandwidth B_{FM} and B_{PM} for the modulating signal $m(t)$ shown below. Assume $k_f = \pi \times 10^4$ rad/s/V and $k_p = \pi/4$ rad/V.
- *Answers*: B_{FM} = 60 kHz; B_{PM} = 40 kHz.



Bandwidth: Example 2

- Estimate the bandwidth B_{FM} and B_{PM} for the modulating signal $m(t)$ shown below. Assume $k_f = \pi \times 10^5$ rad/s/V and $k_p = 5\pi$ rad/V.
- *Answers*: 220 kHz; 20 kHz; 204 kHz; 4 kHz;





FM Signal-to-Noise Ratio

$$SNR_{out} = \left(\frac{3\beta^2}{k_m^2}\right)\frac{S_{in}}{N_0 B}$$

$$S_{in} = \overline{\varphi^2(t)} = \frac{A^2}{2}$$

$$k_m^2 = \frac{m_p^2}{\overline{m^2(t)}}$$

FM (and PM) vs. AM

- FM (and PM) Advantages:
  - FM is capable of exchanging SNR for bandwidth.
  - The constant amplitude of FM makes it less susceptible to nonlinearities.
  - Due to the constant amplitude, FM is less vulnerable than AM to adjacent-channel interference.
  - The constant amplitude of FM gives it a kind of immunity against rapid fading (even with the larger bandwidth).

FM (and PM) vs. AM

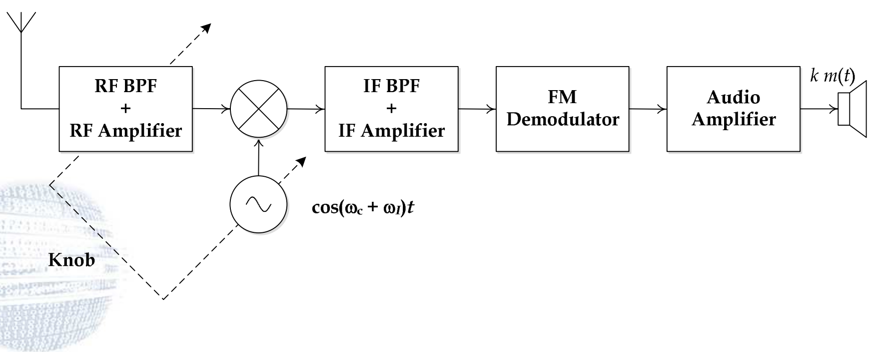
- FM (and PM) Disadvantages:
  - WBFM (which provides better quality) requires large transmission bandwidth.
  - FM modulators and demodulators are relatively more expensive that AM hardware.
  - PM demodulation requires synchronous detection (relatively expensive).

Applications: FM Radio

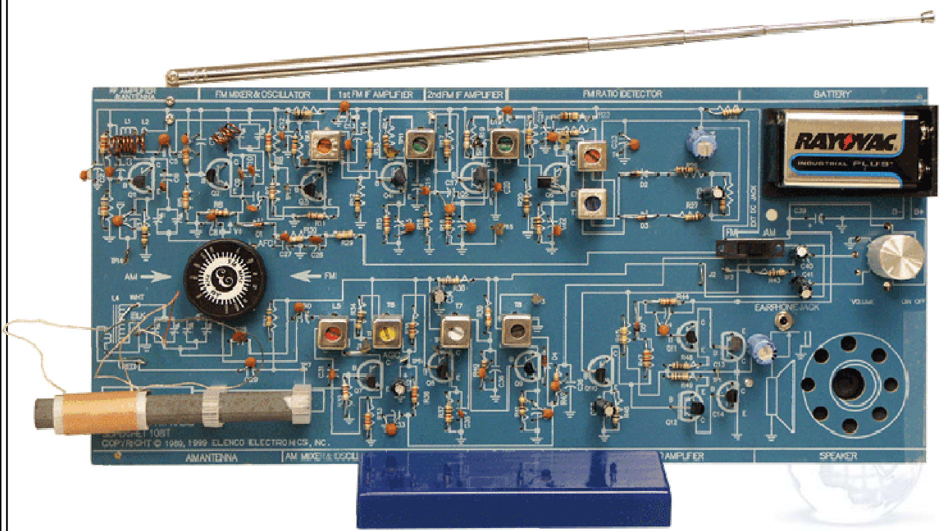
- FM + FDM
  - The baseband message is 15 kHz (voice + music).
  - With β = 5, the bandwidth of each FM station is 200 kHz (both U.S. and Europe).
  - The broadcast range is 88 – 108 MHz.
- FM radio sounds better than AM radio:
  - m(t) has a larger bandwidth.
  - WBFM: exchanging SNR for bandwidth.
  - Pre-emphasis/De-emphasis improves SNR.
  - Stereo FM.

FM Superhetrodyne Receiver

- IF frequency = 10.7 MHz
- L.O. frequency = 88 + 10.7 MH to 108 + 10.7 MHz



FM Superhetrodyne Receiver



Stereo FM vs. Mono FM