Lecture 3: Review of Signal Analysis Basics

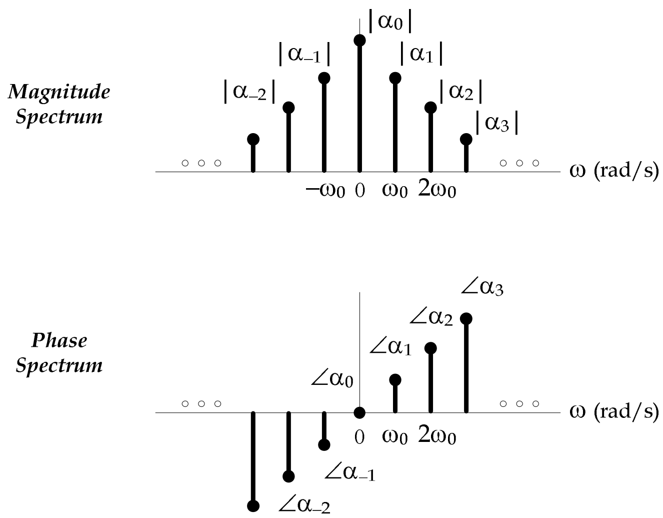
Dr. Mohammed Hawa
Electrical Engineering Department
University of Jordan

Exponential vs. Compact

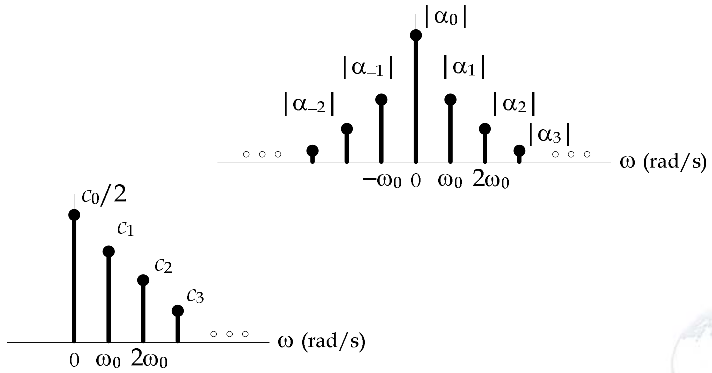
$$x(t) = \sum_{n=-\infty}^{\infty} \alpha_n \cdot e^{jn\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T}$$

$$x(t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t - \theta_n), \quad \omega_0 = \frac{2\pi}{T}$$

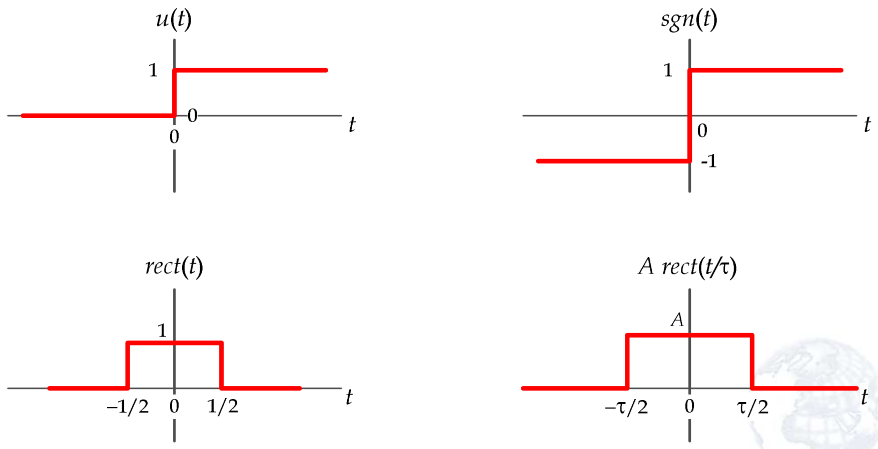
Exponential Fourier Series



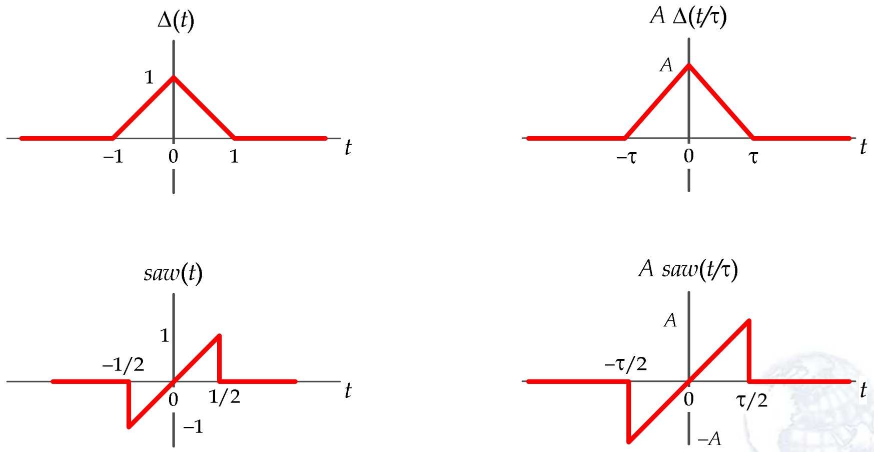
Single-Sided vs. Double-Sided



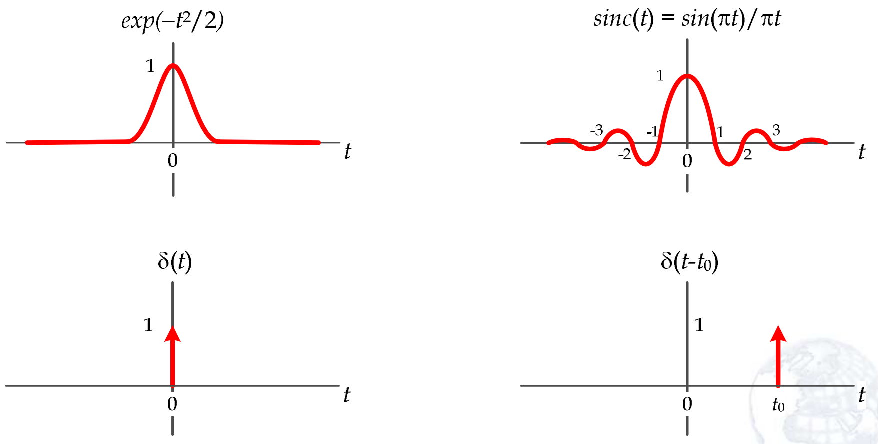
Basic Signals



Basic Signals

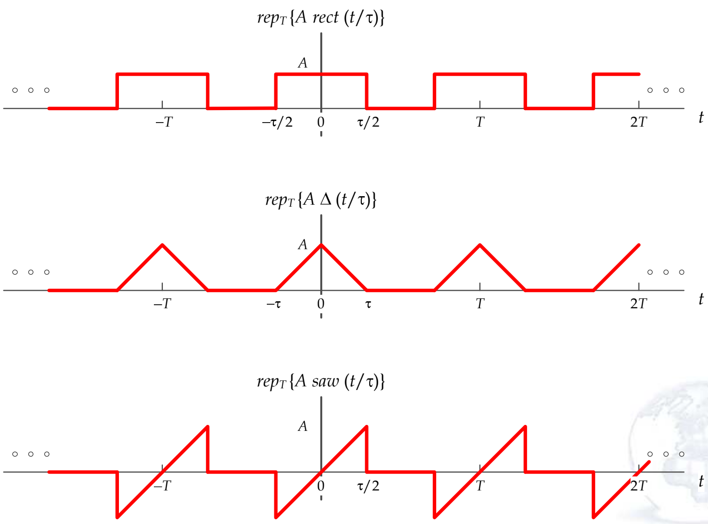


Basic Signals



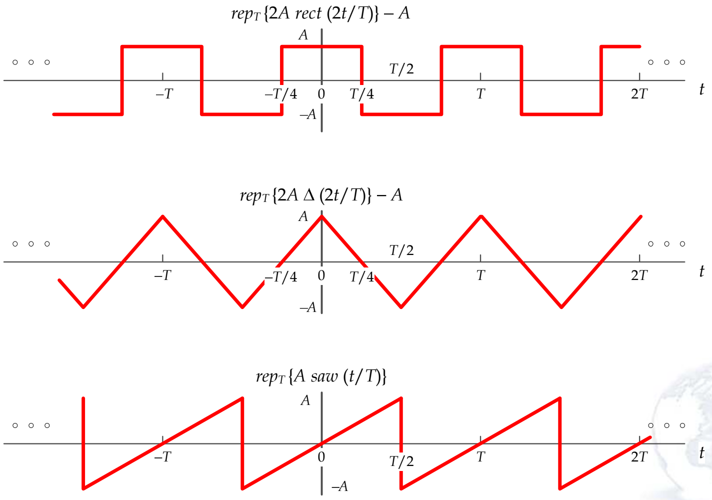
 Electrical Engineering Department, University of Jordan 7

Periodic Signals

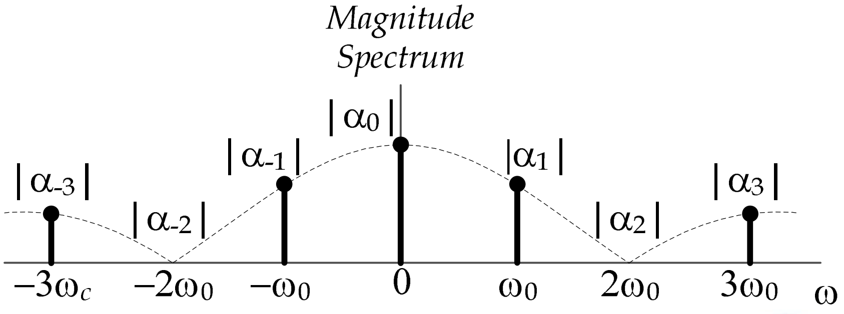


 Electrical Engineering Department, University of Jordan 8

Periodic Signals



First-Null Bandwidth



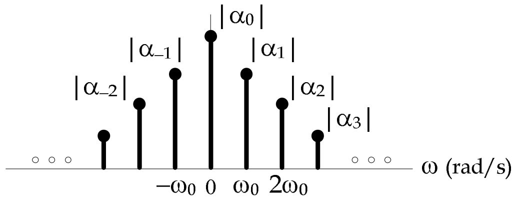
Fourier Transform

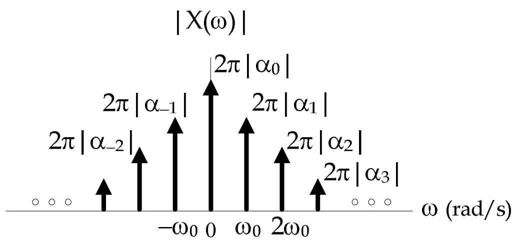
$$X(\omega) = \mathscr{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = \mathscr{F}^{-1}\{X(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

 Electrical Engineering Department, University of Jordan

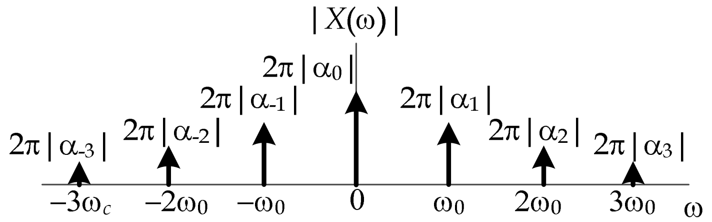
Fourier Series vs. Tranform

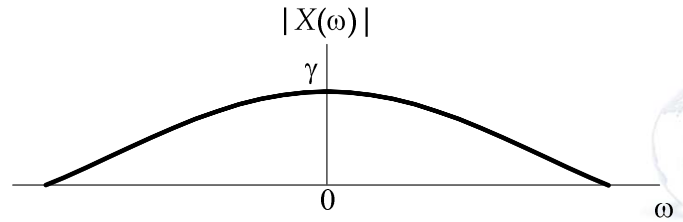




 Electrical Engineering Department, University of Jordan

Periodic vs. Aperiodic





DC vs. Average Power

The DC value or average value of the signal $x(t)$ is:

$$DC = \overline{x(t)} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)dt$$

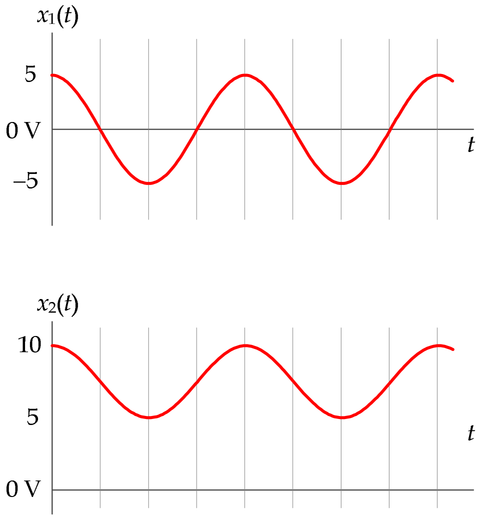
$$DC = \overline{x(t)} = \alpha_0$$

The average power in the signal $x(t)$ is:

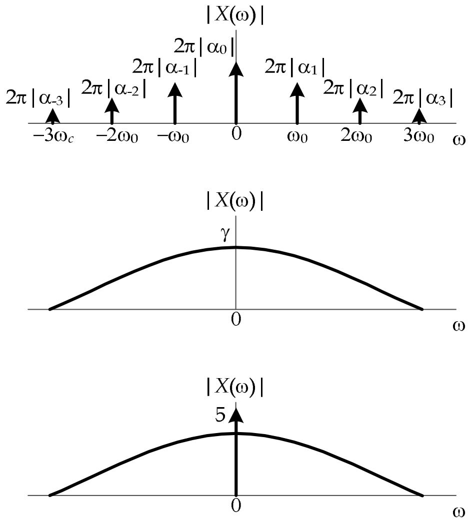
$$P_x = \overline{x^2(t)} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$P_x = \overline{x^2(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega)d\omega$$

DC vs. Average Power

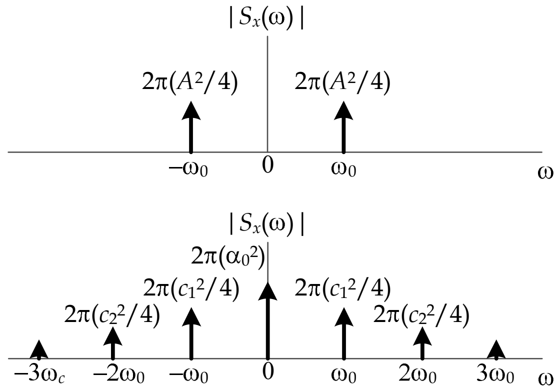


DC from Frequency Domain



Power Spectral Density

$$\text{PSD} = S_x(\omega) = \lim_{T\to\infty} \frac{1}{T} |X_T(\omega)|^2 = \mathcal{F}\{R_{xx}(\tau)\}$$

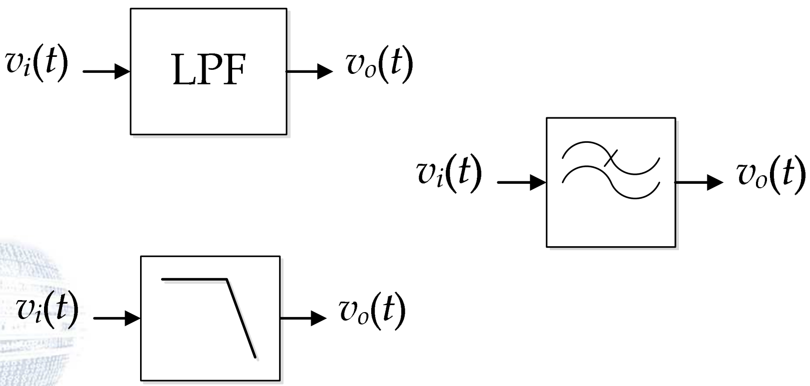


Quick Review of Filters

- There are three main filter types that you studied in signal analysis:
  - LPF: Low-Pass Filter
  - BPF: Band-Pass Filter
  - HPF: High-Pass Filter

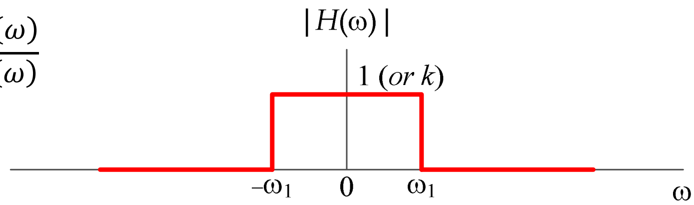
Low-Pass Filter (LPF)

- Symbol:

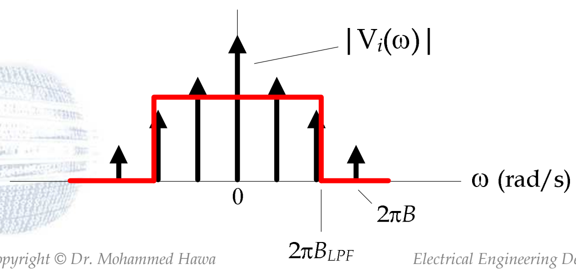


Frequency-response function

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$



$|V_o(\omega)| = |H(\omega)| \times |V_i(\omega)|$



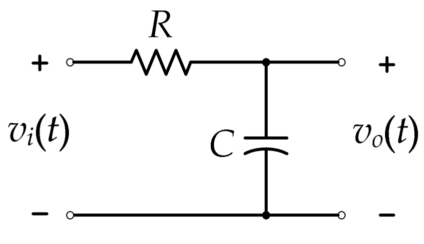
Characteristics

- Always centered at 0 rad/s.
- Bandwidth = Cut-off frequency = ω_1 rad/s
- Gain = k.

$$v_i(t) \rightarrow \boxed{\text{LPF}} \rightarrow v_o(t)$$

w/ Bandwidth = 5 kHz
& Gain = 2

Example Circuit

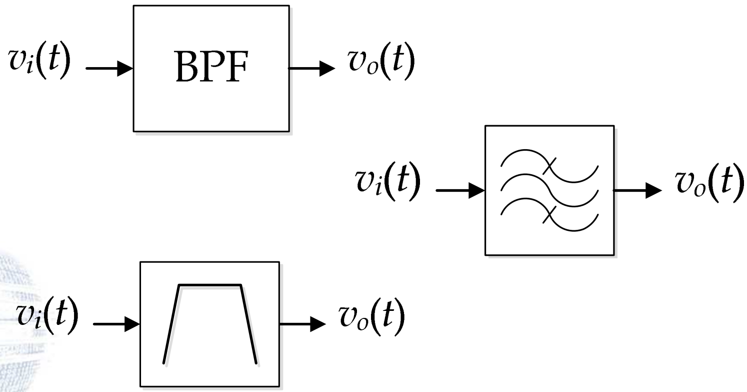


$$B_{LPF} = \frac{1}{2\pi RC}\ \text{Hz}$$

$$Gain = 1$$

Band-Pass Filter (BPF)

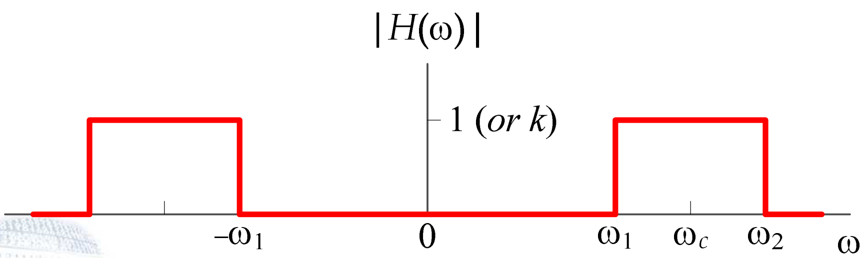
- Symbol:



 Electrical Engineering Department, University of Jordan

Frequency-response function

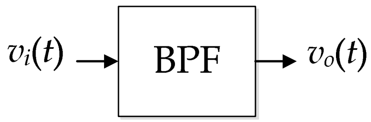
$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$



 Electrical Engineering Department, University of Jordan

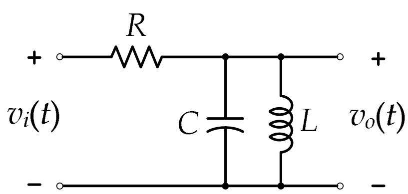
Characteristics

- Centered around center frequency ω_c rad/s.
- Bandwidth of Filter = ω_2 - ω_1 rad/s
- Gain = k.



w/ Bandwidth = 80 kHz
Center Frequency = 100 MHz
& Gain = 1

Example Circuit



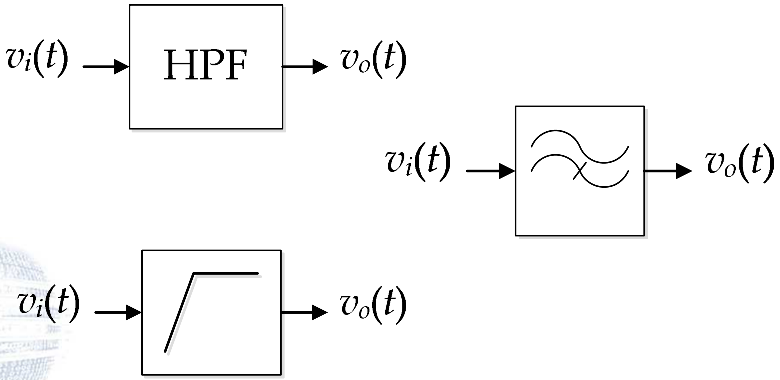
$$f_c = f_{res} = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

$$B_{BPF} = \Delta f = \frac{R}{2\pi L} \text{ Hz}$$

$$Gain = 1$$

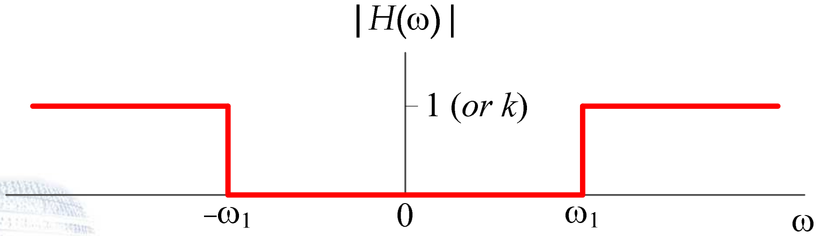
High-Pass Filter (HPF)

- Symbol:



Frequency-response function

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

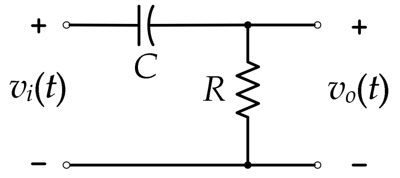


Characteristics

- Cut-off frequency = ω_1 rad/s.
- Gain = k.

- No *bandwidth* defined.

Example Circuit



$$f_{cut-off} = \frac{1}{2\pi RC} \text{ Hz}$$

$$Gain = 1$$