Lecture 3: Review of Signal Analysis Basics

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EE421: Communications I

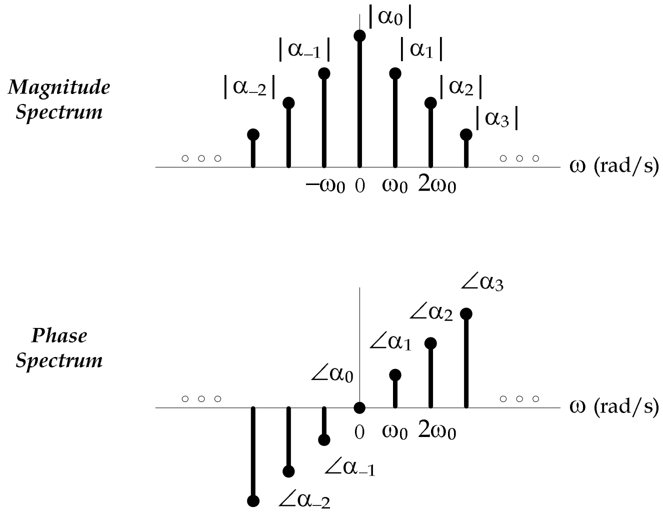
Exponential/Trigonometric/Compact

$$x(t) = \sum_{n=-\infty}^{\infty} \alpha_n \cdot e^{jn\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T}$$

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)), \quad \omega_0 = \frac{2\pi}{T}$$

$$x(t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t - \theta_n), \quad \omega_0 = \frac{2\pi}{T}$$

Exponential Fourier Series



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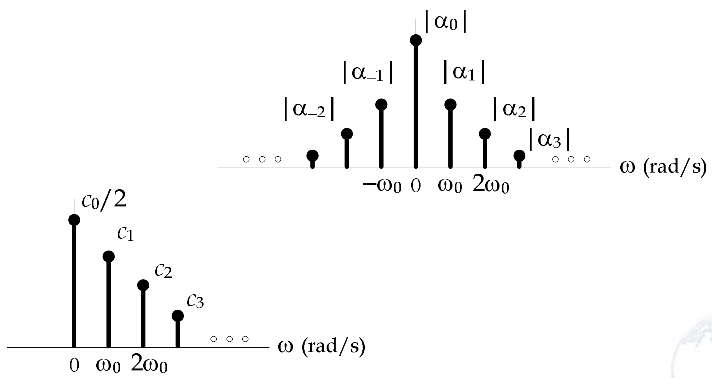
Single-Sided vs. Double-Sided



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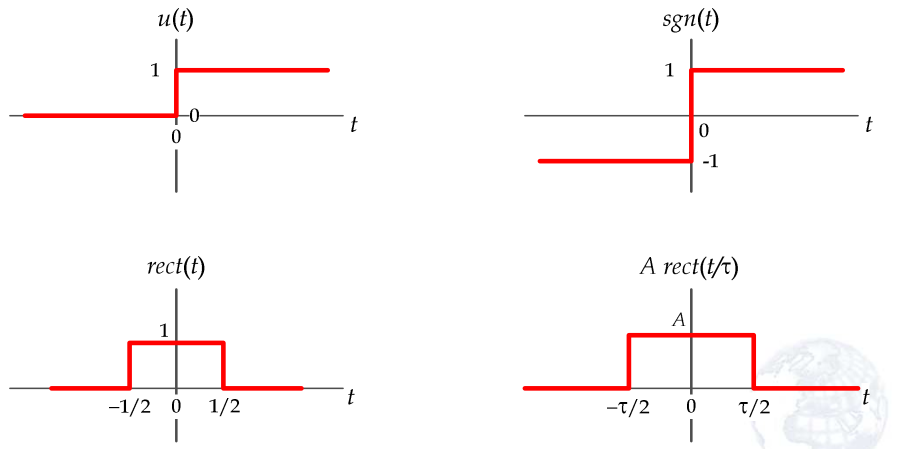
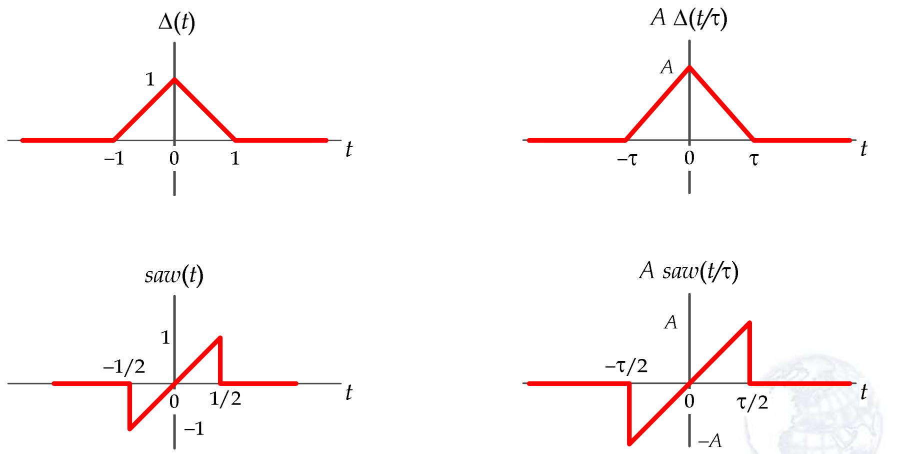
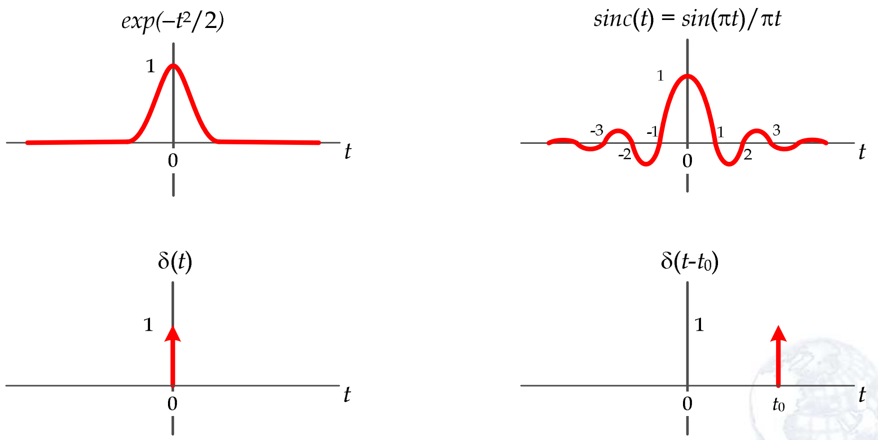
Basic Signals



Basic Signals



Basic Signals



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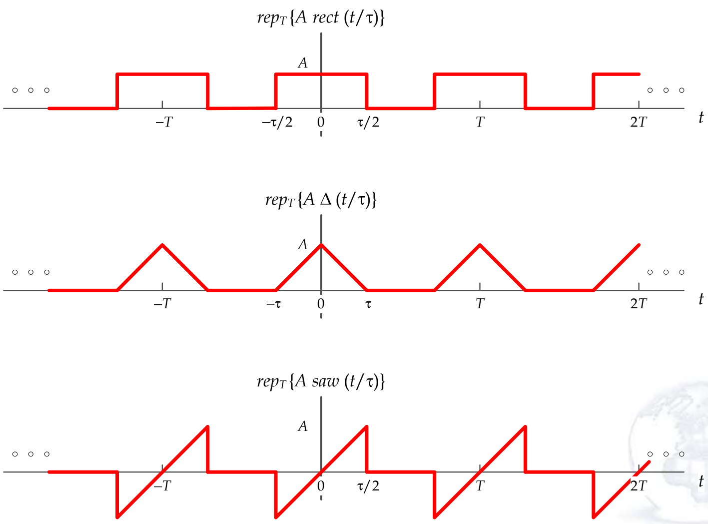
Periodic Signals



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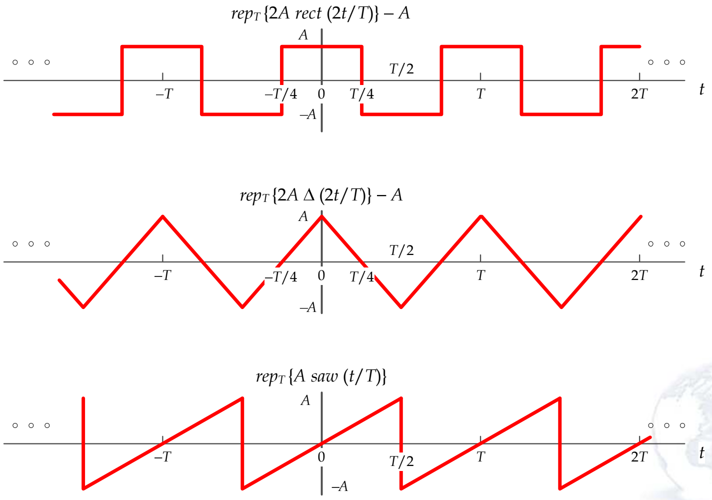
Periodic Signals



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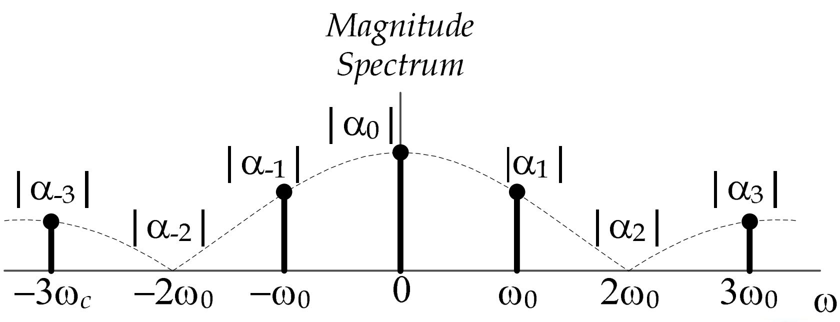
First-Null Bandwidth



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Fourier Transform

$$X(\omega) = \mathscr{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = \mathscr{F}^{-1}\{X(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

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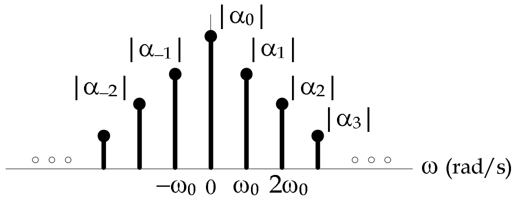
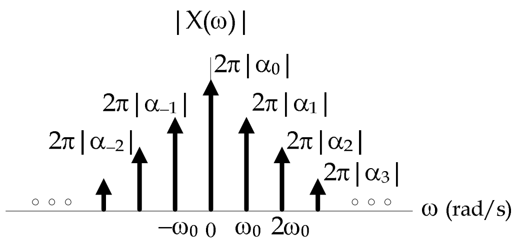
Fourier Series vs. Tranform





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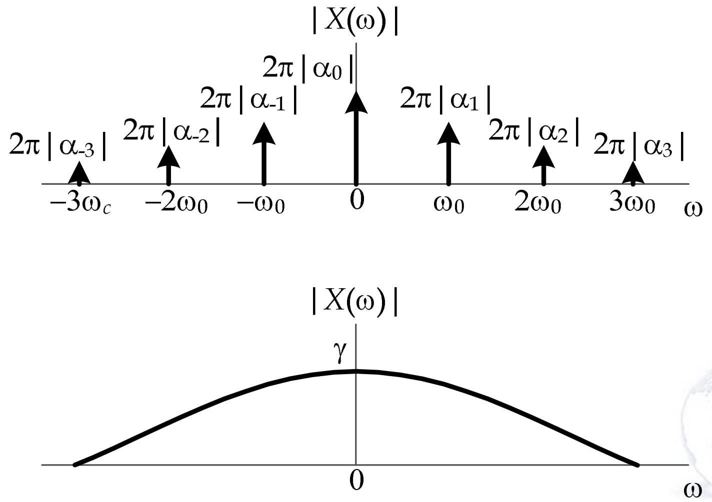
Periodic vs. Aperiodic



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DC vs. Average Power

The DC value or average value of the signal $x(t)$ is:

$$DC = \overline{x(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)dt$$

$$DC = \overline{x(t)} = \alpha_0$$

The average power in the signal $x(t)$ is:

$$P_x = \overline{x^2(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$P_x = \overline{x^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_x(\omega)d\omega$$

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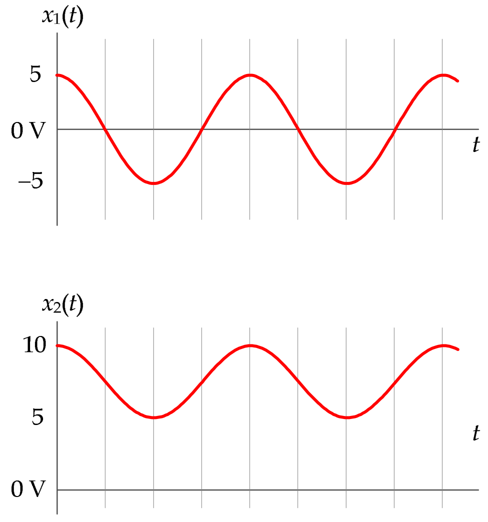
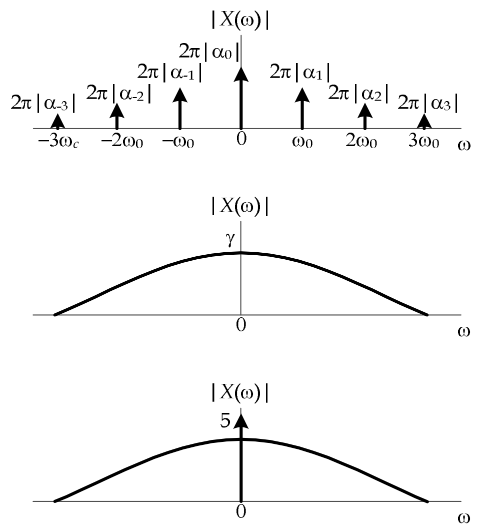
DC vs. Average Power

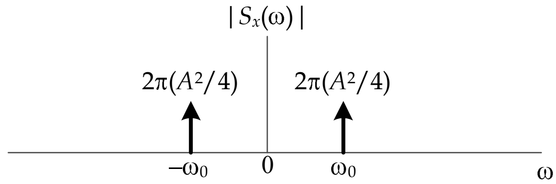


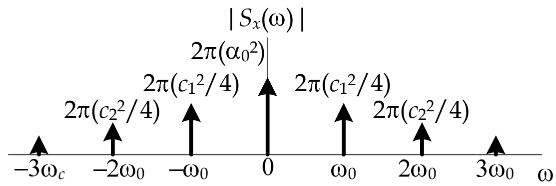
DC from Frequency Domain



Power Spectral Density

$$\text{PSD} = S_x(\omega) = \lim_{T\to\infty} \frac{1}{T} |X_T(\omega)|^2 = \mathcal{F}\{R_{xx}(\tau)\}$$





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Quick Review of Filters

- There are four main filter types that you studied in signal analysis:
  - **LPF: Low-Pass Filter**
  - **BPF: Band-Pass Filter**
  - HPF: High-Pass Filter
  - Band-Stop Filter / Notch Filter.

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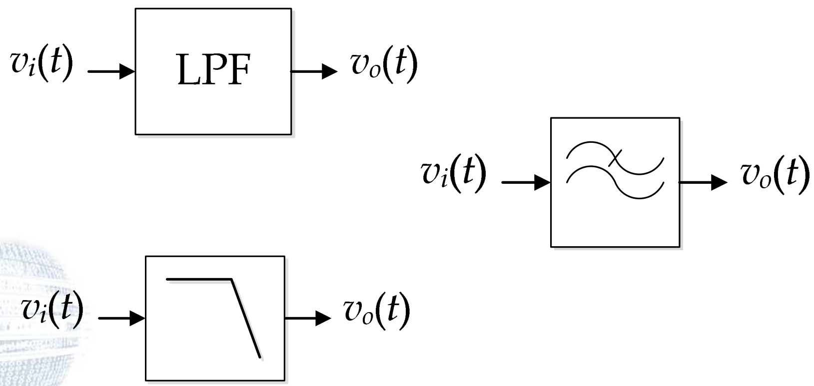
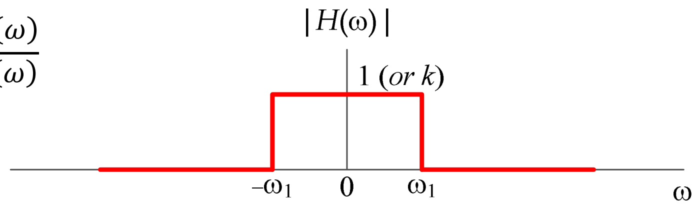
Low-Pass Filter (LPF)

- Symbol:

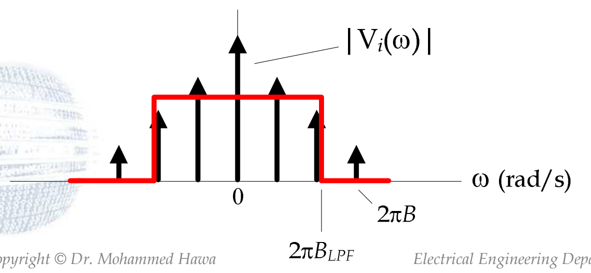


Frequency-response function

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

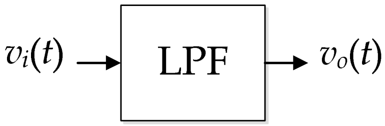


$|V_o(\omega)| = |H(\omega)| \times |V_i(\omega)|$



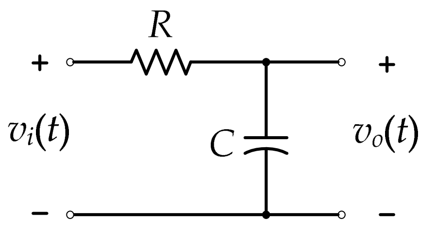
Characteristics/Specifications

- Always centered at 0 rad/s.
- Bandwidth = Cut-off frequency = ω_1 rad/s
- Gain = k.



w/ Bandwidth = 5 kHz
& Gain = 2

Example Circuit

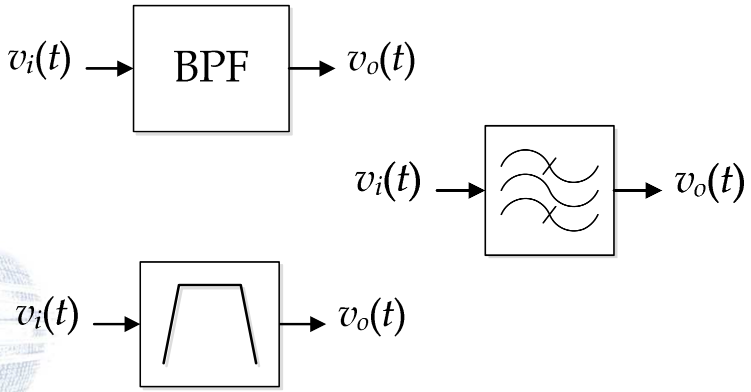


$$B_{LPF} = \frac{1}{2\pi RC}\ \text{Hz}$$

$$Gain = 1$$

Band-Pass Filter (BPF)

- Symbol:



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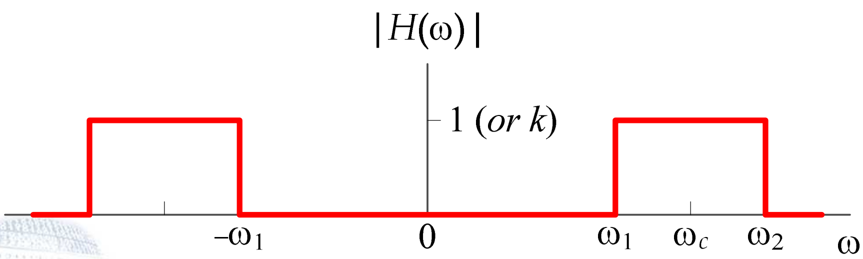
Frequency-response function

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$



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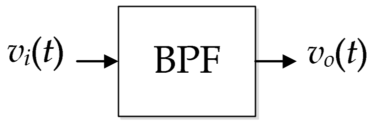
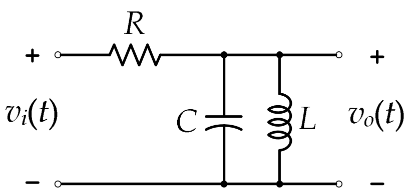
Characteristics/Specifications

- Centered around center frequency ω_c rad/s.
- Bandwidth of Filter = $\omega_2 - \omega_1$ rad/s
- Gain = k.



w/ Bandwidth = 80 kHz
Center Frequency = 100 MHz
& Gain = 1

Example Circuit



$$f_c = f_{res} = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

$$B_{BPF} = \Delta f = \frac{R}{2\pi L} \text{ Hz}$$

$$Gain = 1$$