Lecture 8: Performance of Communication Systems

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EE421: Communications I. For more information read Section 9.9 in your textbook or visit http://wikipedia.org/.

Shannon's Limit

$$C = B_{ch} \times log_2(1 + SNR)$$

Shannon's Limit:
- C: Capacity of the channel in bits/second (bps)
- B_{ch}: Channel bandwidth (units of Hz)
- SNR: Signal-to-Noise Ratio (unitless)

Signal-to-Noise Ratio (SNR)

$$SNR = \frac{Signal\ \ Power}{Noise\ \ Power} = \frac{P_x}{P_n} \qquad (unitless)$$

- **Noise:** all random and unpredictable signals added to the message by the channel.
- External and internal sources of noise.
- Solutions exist to reduce noise power (but noise can *never* be eliminated).
- In analog systems, SNR decides the *Quality* of the received *noisy* signal.
- In digital systems, SNR decides *BER*, i.e., *Quality*.

Quality

SNR is usually expressed in dB

$$SNR = \frac{Signal\ \ Power}{Noise\ \ Power} = \frac{P_x}{P_n} \quad (unitless)$$

$$SNR = 10 \times \log_{10}\left(\frac{P_x}{P_n}\right) \quad (dB)$$

$$\frac{P_2}{P_1}[dB] = 10 \times \log_{10}\left(\frac{P_2}{P_1}[unitless]\right) = 10 \times \log_{10}\left(\left(\frac{V_2}{V_1}\right)^2 [unitless]\right)$$

$$\frac{V_2}{V_1}[dB] = 20 \times \log_{10}\left(\frac{V_2}{V_1}[unitless]\right)$$

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Unitless vs. dB

$$\frac{P_2}{P_1}[dB] = 10 \times \log_{10}\left(\frac{P_2}{P_1}[unitless]\right)$$

$$\frac{P_2}{P_1}[unitless] = 10^{\left[\frac{\frac{P_2}{P_1}[dB]}{10}\right]}$$

$$1000\ unitless \Rightarrow 10 \times \log_{10}(1000) = 30\ dB$$

$$30\ dB \Rightarrow 10^{\left[\frac{30}{10}\right]} = 10^3 = 1000\ unitless$$

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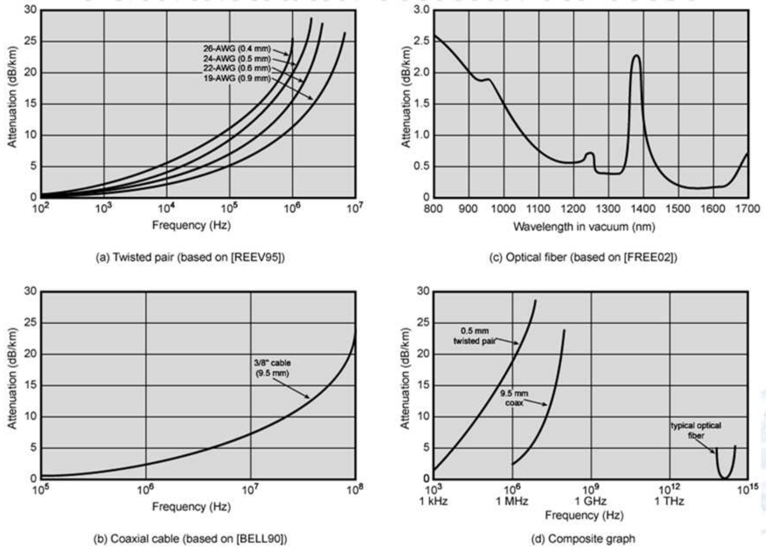
Gain, Power	Gain in dB
1 (no gain)	0 dB
2 (twice the power)	≈ +3 dB
10 (ten times the power)	+10 dB
100	+20 dB
1000	+30 dB
10000	+40 dB

Attenuation, Power	Attenuation in dB
0.5 (half the power)	≈ -3 dB
0.25 (quarter the power)	≈ -6 dB
0.1 (tenth the power)	-10 dB
0.01 (one hundredth)	-20 dB
0.001 (one in a thousand)	-30 dB
0.0001 (one in 10 thousand)	-40 dB

Datasheets (units of dB)



dB, dBm and dBW

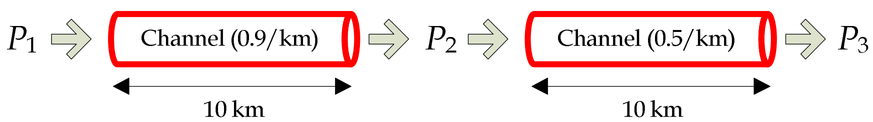
$$\frac{P_2}{P_1}[unitless] \rightarrow \frac{P_2}{P_1}[dB] = 10 \times \log_{10}\left(\frac{P_2}{P_1}[unitless]\right)$$

$$P_2[mW] \rightarrow P_2[dBm] = 10 \times \log_{10}\left(\frac{P_2[mW]}{1\ mW}\right)$$

$$P_2[W] \rightarrow P_2[dBW] = 10 \times \log_{10}\left(\frac{P_2[W]}{1\ W}\right)$$

Example

- Assume P_1 is 100 mW. Find P_3 in dBm.

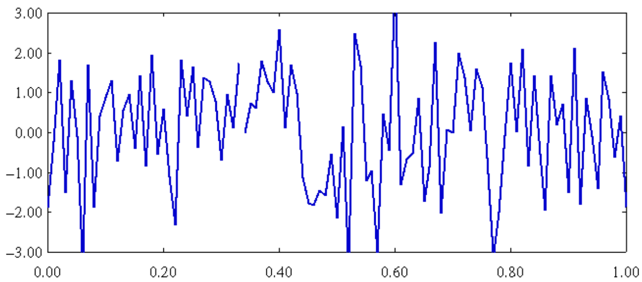


SNR vs. Quality

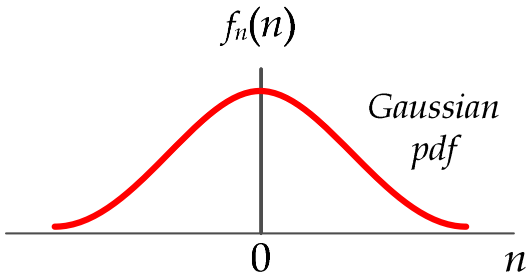
- For **voice** signals:
  - SNR = 5 dB to 10 dB at the receiver output implies a barely intelligible signal.
  - SNR = 25 dB to 35 dB is telephone quality signal.
  - **Summary:** If SNR ≥ 30 dB, good quality voice.
- For **video** signals:
  - **Summary:** If SNR ≥ 50 dB, good quality video.
- For **digital** signals:
  - Need enough SNR for BER ≤ 10^{-6}, good quality.

Noise in Time Domain

- Noise is a purely **random** signal.
- Cannot be written as a deterministic equation.



Noise in Time Domain



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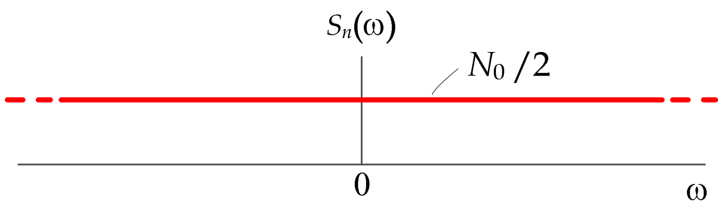
Noise in Frequency Domain



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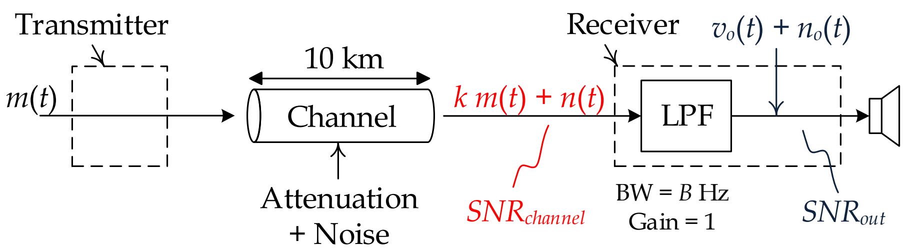
Example

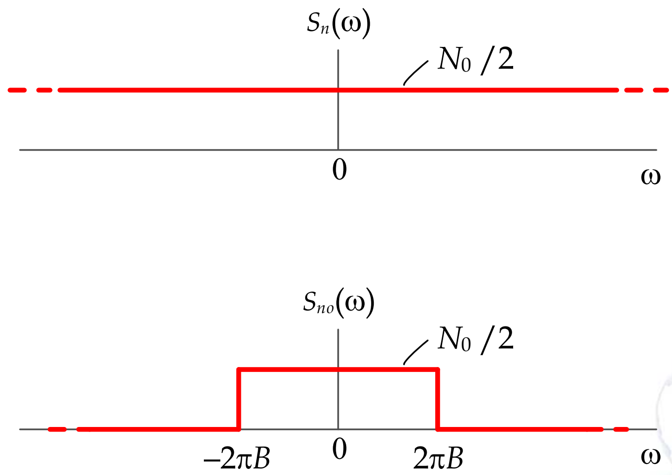
- A voice signal $m(t)$ is transmitted without modulation through a 10 km long baseband channel with AWGN noise. Assume:
  - Average power in $m(t)$ at the TX is 1 kW
  - Channel Attenuation = –5 dB/km
  - $S_n(\omega) = 2 \times 10^{-9}$ W/Hz $= N_0 / 2$
- Show the block diagram of the receiver.
- Determine $SNR_{channel}$ and SNR_{out}.

Solution



$$SNR_{channel} = \frac{k^2\overline{m^2(t)}}{\overline{n^2(t)}} \qquad SNR_{out} = \frac{\overline{v_o^2(t)}}{\overline{n_o^2(t)}}$$

Band-Limited Noise



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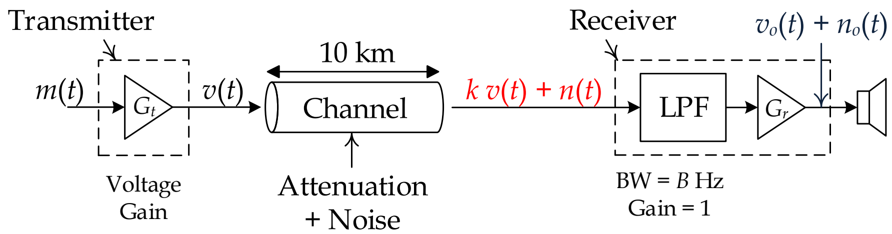
Homework



- Is the gain G_r useful for improving quality?
- Is the gain G_r useful for anything else?
- Determine the gain G_t at the transmitter to get good quality voice at the receiver output.
- *Or* determine the necessary cooling (temperature) of the channel to get good quality voice.
- *Or* determine the channel attenuation to get good quality voice.

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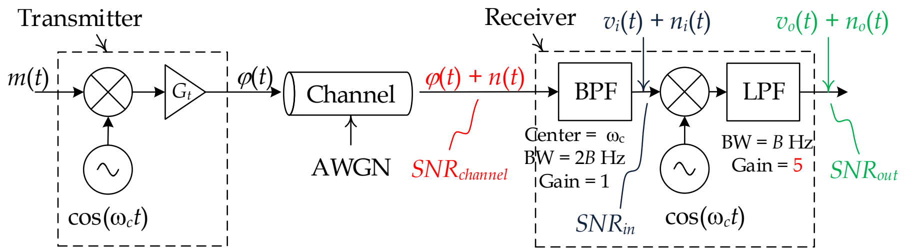
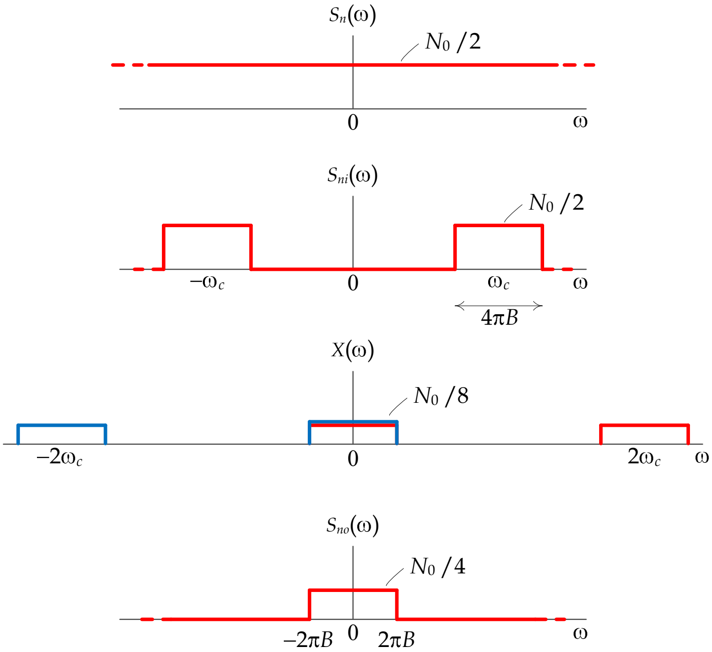
Example 2

- A DSB-SC signal is sent though a channel with no attenuation. The channel is affected by AWGN noise. At the receiver side:
- Show the block diagram of the receiver.
- Determine $SNR_{channel}$.
- Determine SNR_{in}.
- Determine SNR_{out}.
- Determine NF for the demodulator.

Solution



$$SNR_{channel} = 0\ ,\ SNR_{in} = \frac{\overline{m^2(t)}}{4N_0B}\ ,\quad SNR_{out} = \frac{\overline{m^2(t)}}{2N_0B}$$



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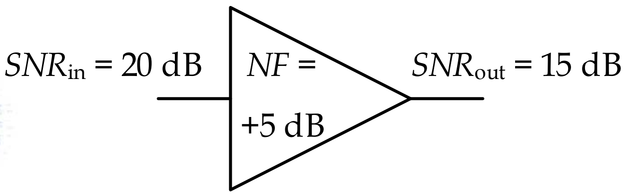
Noise Figure, *NF*

- A number by which the performance of a device can be specified. It measures the *degradation* of the quality (SNR) caused by components in this device.

$$NF \triangleq SNR_{in}(dB) - SNR_{out}(dB)$$



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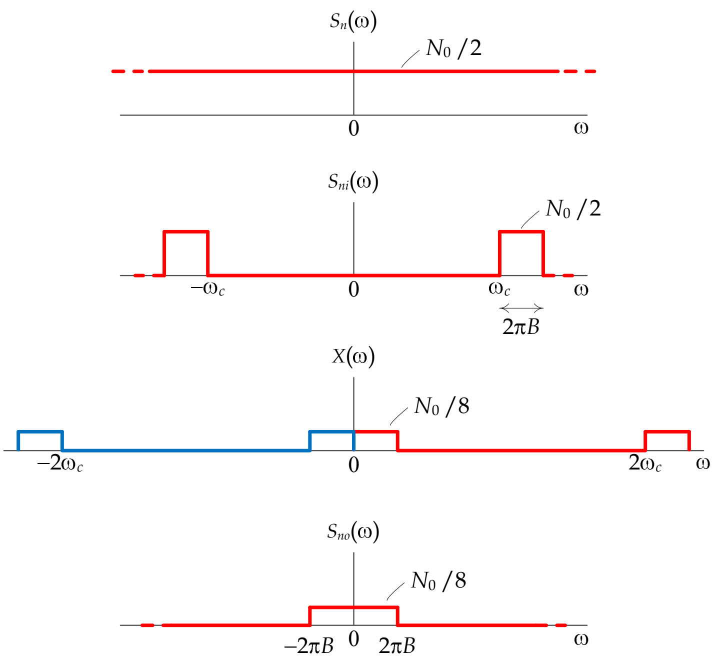
Homework 1

- A SSB-SC (USB) signal is sent though a channel with no attenuation. The channel is affected by AWGN noise. At the receiver side:
- Show the block diagram of the receiver.
- Determine $SNR_{channel}$.
- Determine SNR_{in}.
- Determine SNR_{out}.
- Determine NF for the demodulator.



Homework 2

- A QAM signal is sent though a channel with no attenuation. The channel is affected by AWGN noise. Assume that: $\overline{m_1^2} = \overline{m_2^2}$
  At the receiver side:
- Show the block diagram of the receiver.
- Determine $SNR_{channel}$.
- Determine SNR_{in}.
- Determine $SNR_{out,1}$ and $SNR_{out,2}$.
- Determine NF for the demodulator.

Modulation Technique	Modulated Signal Bandwidth	SNR_{out}	Noise Figure NF, dB	Typical Applications
DSB-SC	$2B$	$\frac{S_{in}}{N_0 B}$	-3	Analog instrumentation; multiplexing as part of FM stereo
SSB-SC	B	$\frac{S_{in}}{N_0 B}$	0	Point-to-point voice
VSB-SC	$B{\sim}2B$	$\frac{S_{in}}{N_0 B}$	$-3{\sim}0$	Facsimile (Fax machines)
QAM	$2B$ for two signals	$\frac{S_{in,effective}}{N_0 B}$	0	Transmit color information in TV broadcasting; digital data
AM	$2B$	$\eta \frac{S_{in}}{N_0 B}$	$-10\log(2\eta)$	Broadcast AM radio; point-to-point voice
SSB+C	B	$\eta \frac{S_{in}}{N_0 B}$	$-10\log(\eta)$	Multiplexing in old telephony systems; point-to-point voice
VSB+C	$B{\sim}2B$	$\eta \frac{S_{in}}{N_0 B}$	$-10\log(2\eta) \sim -10\log(\eta)$	Analog Television broadcasting
FM	$2\Delta f + 2B$	$\left(\frac{3\beta^2}{k_m^2}\right)\frac{S_{in}}{N_0 B}$	$10\log\left(\frac{k_m^2}{6(\beta+1)\beta^2}\right)$	Broadcast FM radio; analog microwave links
PM	$2\Delta f + 2B$	$\left(\frac{(\Delta\theta)^2}{k_m^2}\right)\frac{S_{in}}{N_0 B}$	$10\log\left(\frac{k_m^2 B}{2(\Delta\theta)^2(\Delta f + B)}\right)$	Telemetry; digital data