Lecture 7: Frequency & Phase Modulation (FM & PM)

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Angle Modulation (FM & PM)

$$c(t) = A\cos(\omega_c t + \theta_0)$$

$$\varphi_{FM\ or\ PM}(t) = A\cos\theta(t)$$

$$\omega_i(t) \triangleq \frac{d\theta(t)}{dt}$$

$$\theta_i(t) \triangleq \theta(t) - \omega_c t$$

$\theta(t)$: **generalized angle** of modulated signal.
$\omega_i(t)$: **instantaneous frequency** of modulated signal.
$\theta_i(t)$: **instantaneous phase** of modulated signal.

Frequency Modulation (FM)

- The *instantaneous frequency* of the modulated signal changes in proportion to the message.

$$\omega_{i_{FM}}(t) = \omega_c + k_f m(t)$$

$$\theta_{FM}(t) = \omega_c t + k_f \int_{-\infty}^{t} m(t)dt$$

$$\varphi_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(t)dt\right)$$

$$\theta_{i_{FM}}(t) = k_f \int_{-\infty}^{t} m(t)dt$$

Phase Modulation (PM)

- The *instantaneous phase* of the modulated signal changes in proportion to the message.

$$\theta_{i_{PM}}(t) = k_p m(t)$$

$$\theta_{PM}(t) = \omega_c t + k_p m(t)$$

$$\varphi_{PM}(t) = A\cos\left(\omega_c t + k_p m(t)\right)$$

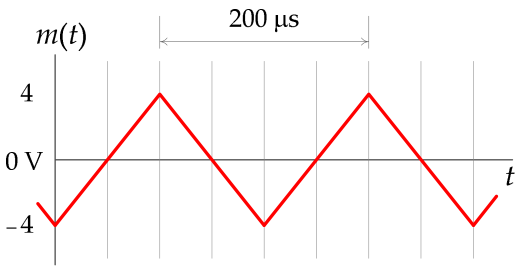
$$\omega_{i_{PM}}(t) = \omega_c + k_P \frac{dm(t)}{dt} = \omega_c + k_P m'(t)$$

FM and PM Equivalence

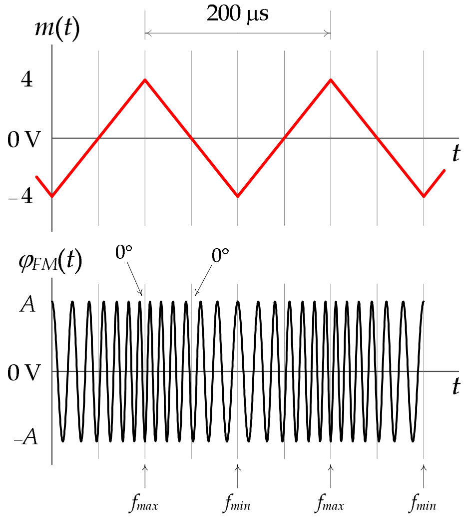
- FM
  - Constant amplitude A
  - Constant carrier frequency ω_c
  - Variable *instantaneous* frequency $\omega_i \propto m(t)$
  - Variable *instantaneous* phase $\theta_i \propto \int m(t)dt$
- PM
  - Constant amplitude A
  - Constant carrier frequency ω_c
  - Variable *instantaneous* frequency $\omega_i \propto m'(t)$
  - Variable *instantaneous* phase $\theta_i \propto m(t)$

Example 1

- For the following message signal $m(t)$ and a 100 MHz carrier:
  a) Sketch the **FM** modulated signal. Use $k_f = 2\pi\times10^5$ rad/s/V.
  b) Sketch the **PM** modulated signal. Use $k_p = 5\pi$ rad/V.
  c) Find Δf for both modulated signals.



Solution: FM



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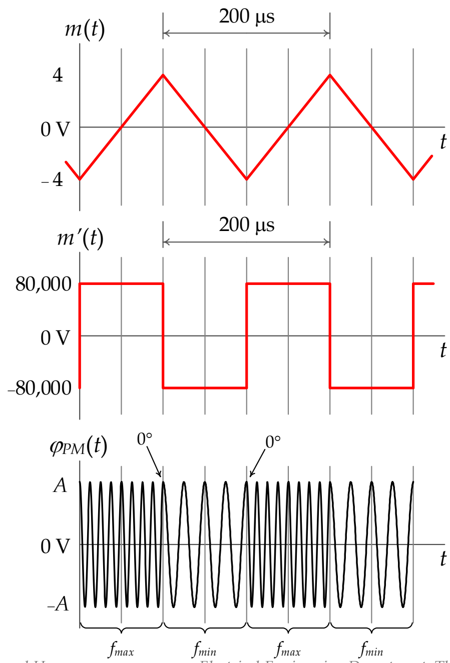
PM



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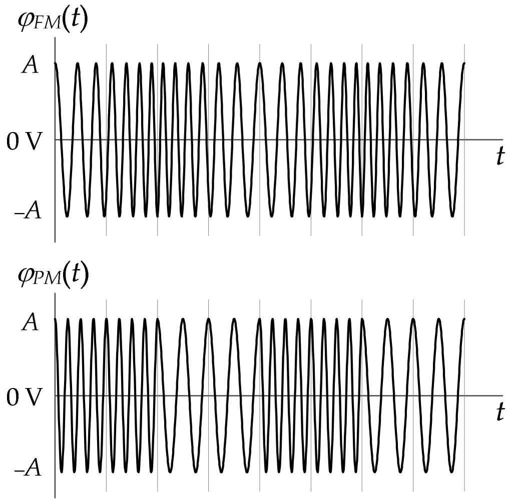
FM vs. PM



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Peak Frequency Deviation

- For **FM**:

$$\Delta f \triangleq \frac{f_{max} - f_{min}}{2} = \frac{k_f}{2\pi} \times \frac{m(t)_{max} - m(t)_{min}}{2}$$

$$\Delta f = \frac{k_f}{4\pi} \times m(t)_{pk-pk} \quad [Hz]$$

- For **PM**:

$$\Delta f \triangleq \frac{f_{max} - f_{min}}{2} = \frac{k_p}{2\pi} \times \frac{m'(t)_{max} - m'(t)_{min}}{2}$$

$$\Delta f = \frac{k_p}{4\pi} \times m'(t)_{pk-pk} \quad [Hz]$$

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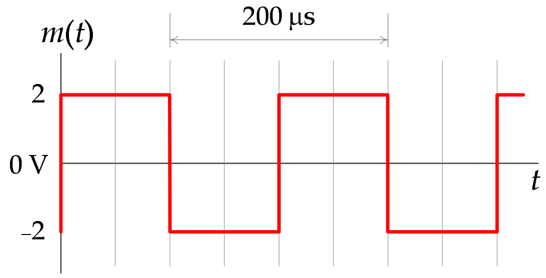
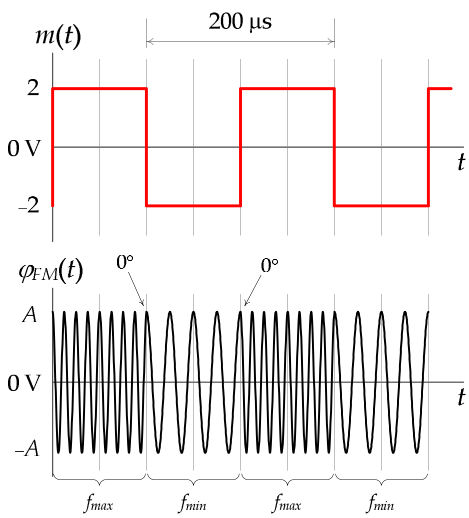
Example 2

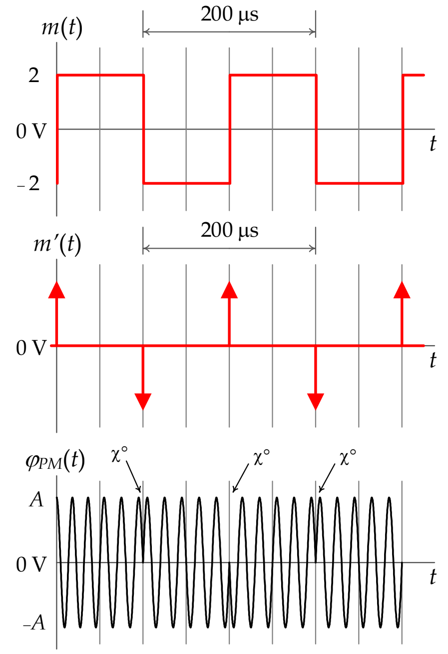
- For the following message signal $m(t)$ and a 100 MHz carrier:
  a) Sketch the **FM** modulated signal. Use $k_f = 2\pi\times10^5$ rad/s/V.
  b) Sketch the **PM** modulated signal. Use $k_p = \pi/4$ rad/V.
  c) Find Δf for both modulated signals.



Solution: FM or FSK



PM or BPSK



Rules of Thumb

- Smooth change in instantaneous frequency *always* means smooth change in instantaneous phase.
- Sudden change in instantaneous frequency (i.e., unit step change) *does not* mean a sudden change in phase, i.e., it means 0° sudden phase shift.
- *Impulse* change in instantaneous frequency (i.e., infinity frequency) *might* cause a sudden change in phase. To determine the sudden phase shift (or lack thereof) see $k_p m(t)$ for PM or $k_f \int m(t)dt$ for FM.

