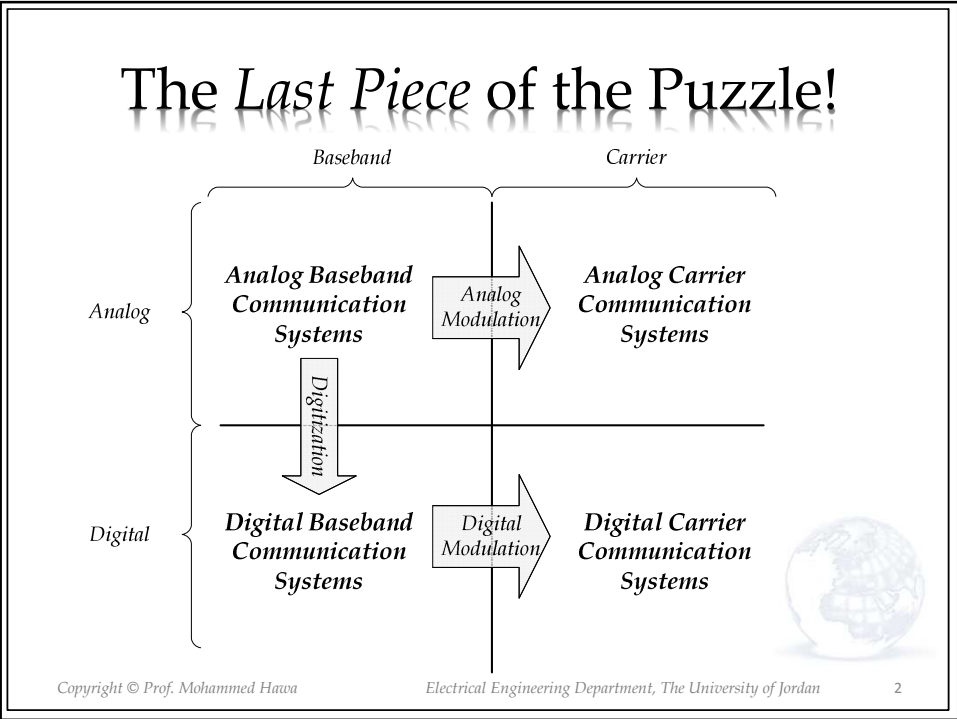


# Lecture 19: Digital Modulation

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EE421: Communications I. For more information read Chapters 7 & 10 in your textbook or visit <http://wikipedia.org/>.



## Digital Modulation

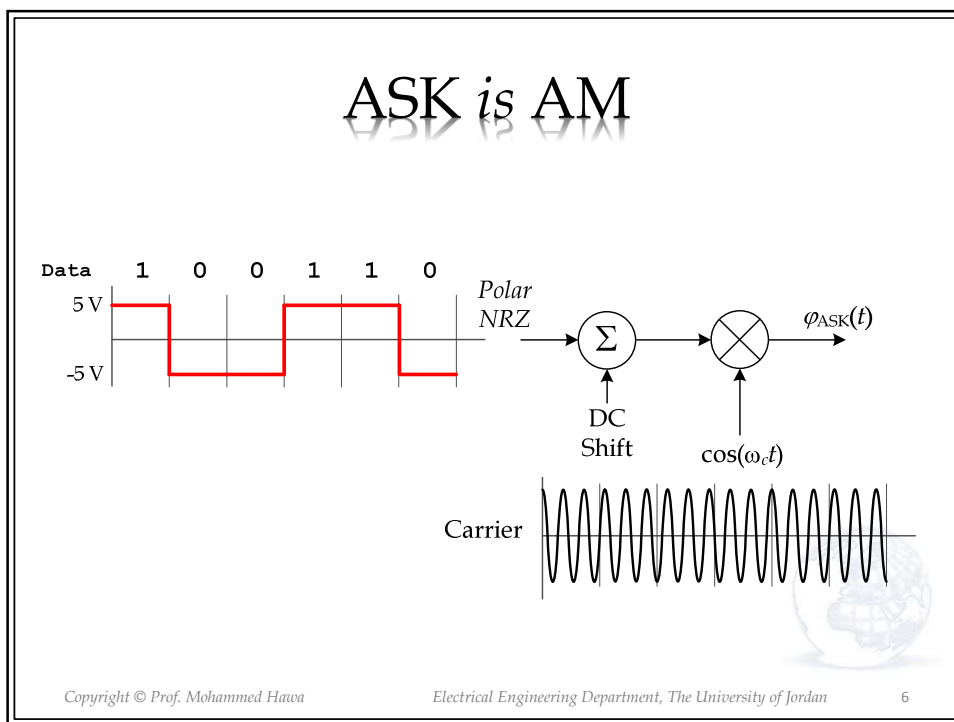
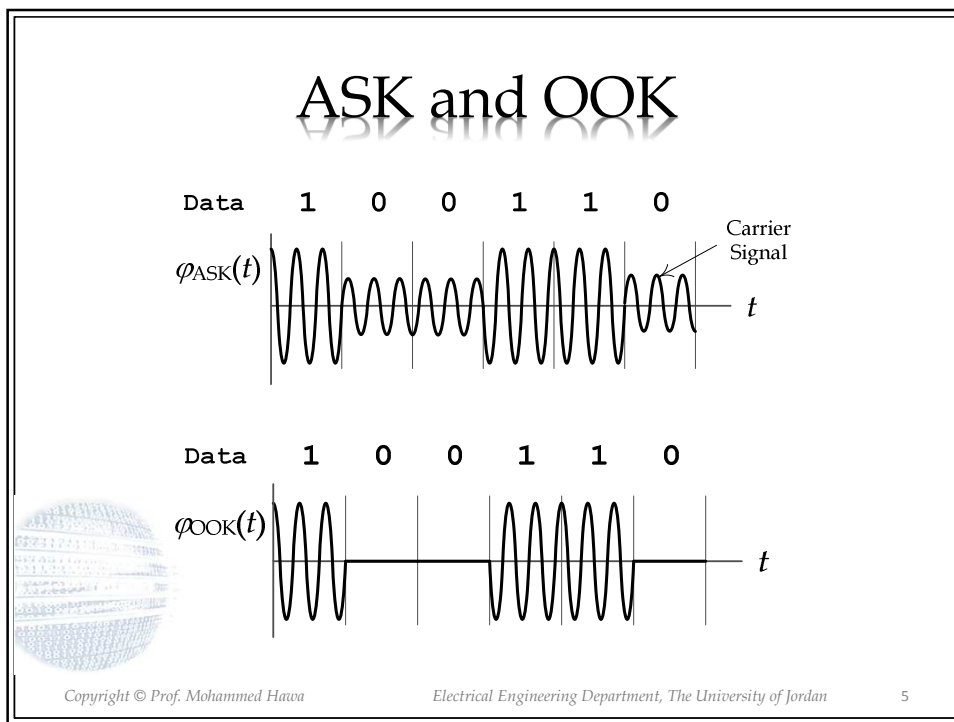
- *Four* main modulation techniques:
  - Amplitude-Shift Keying (**ASK**).
  - Frequency-Shift Keying (**FSK**).
  - Phase-Shift Keying (**PSK**).
  - Quadrature Amplitude Modulation (**QAM**).
- PSK and QAM are the most popular nowadays because of their *smaller* bandwidths.
- PSK and QAM require synchronous detection, which is easier nowadays (PLLs).

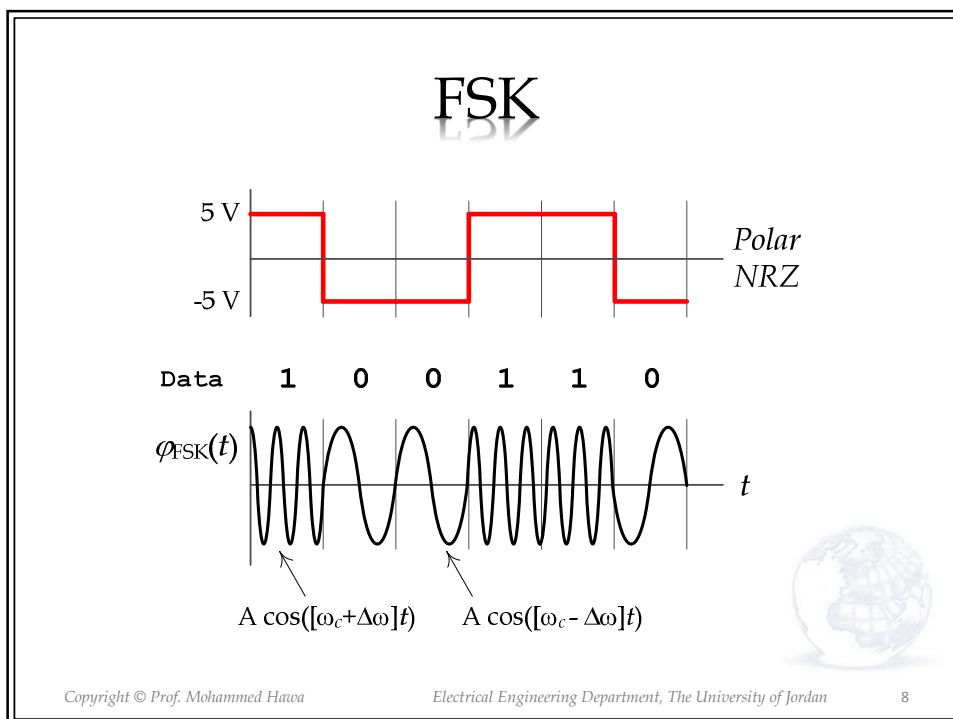
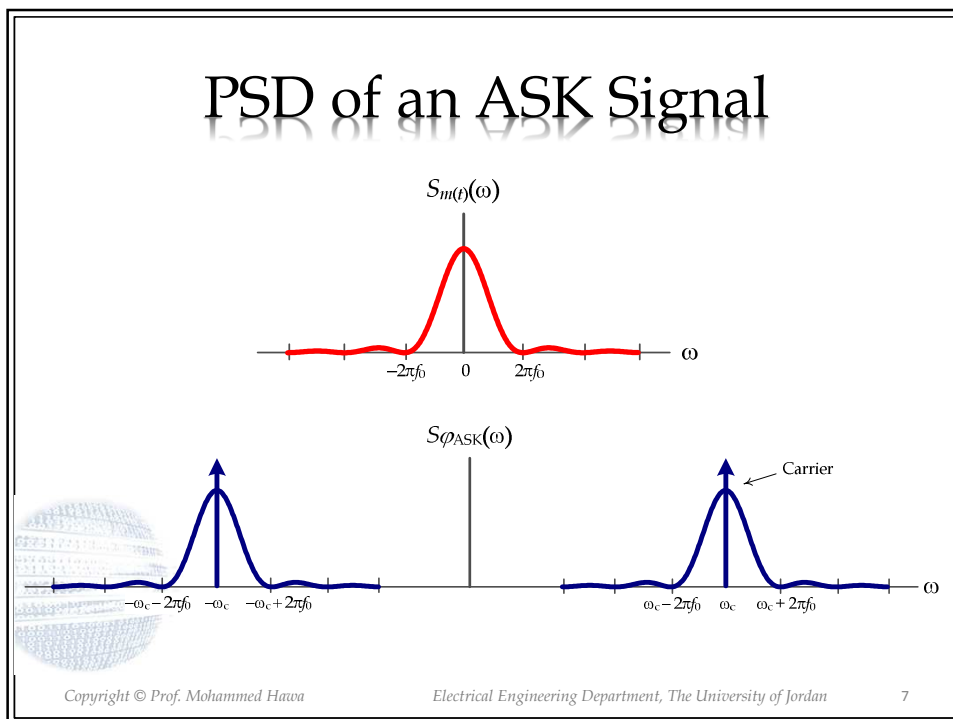


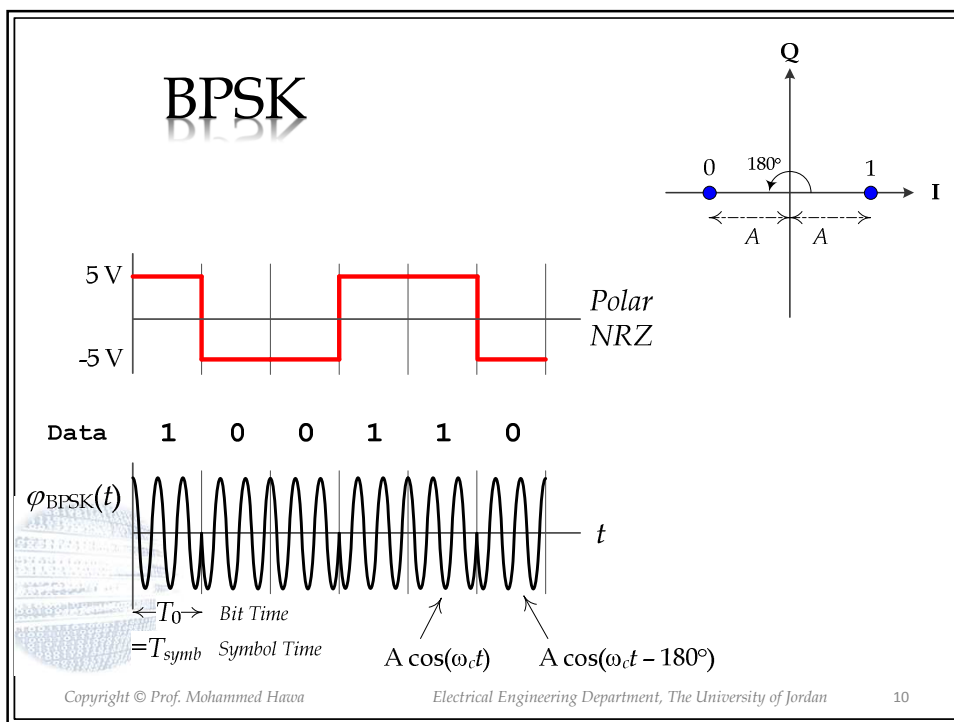
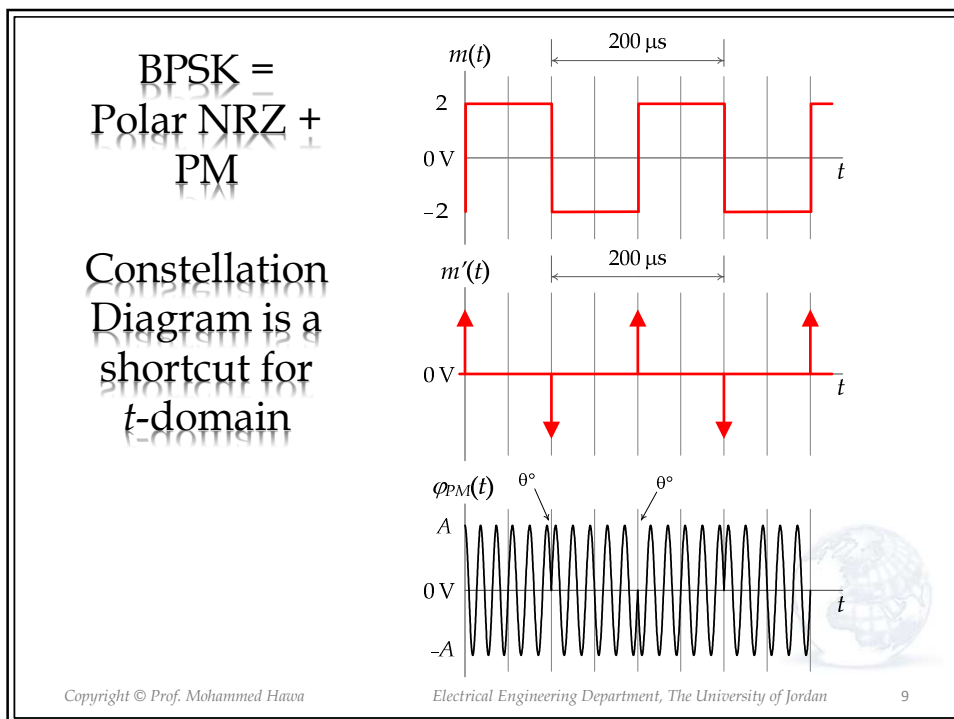
## Analog vs. *Digital* Modulation

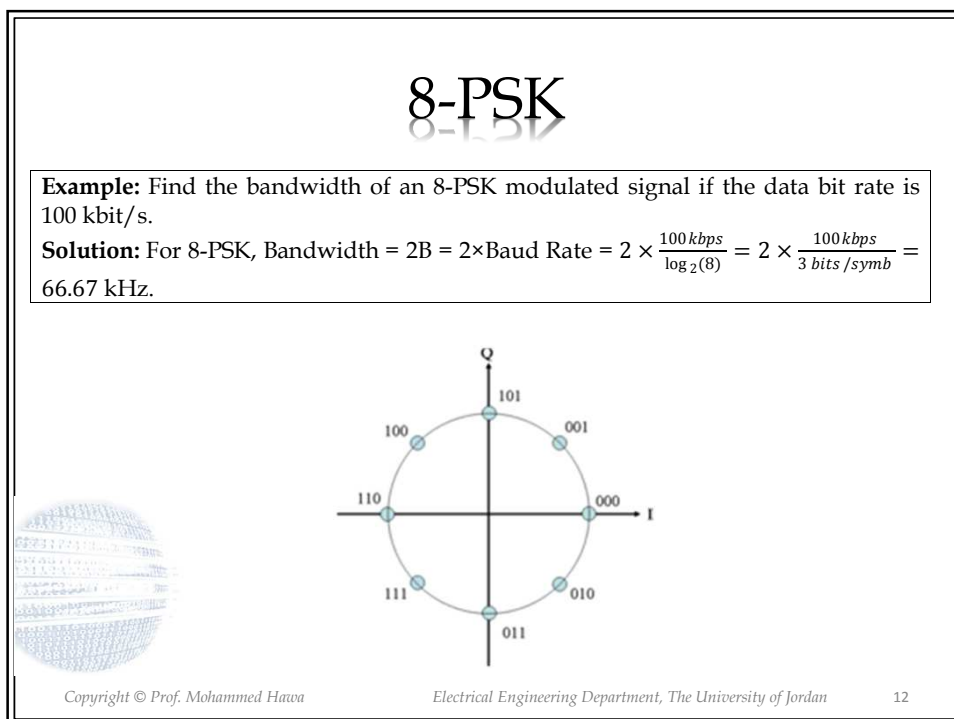
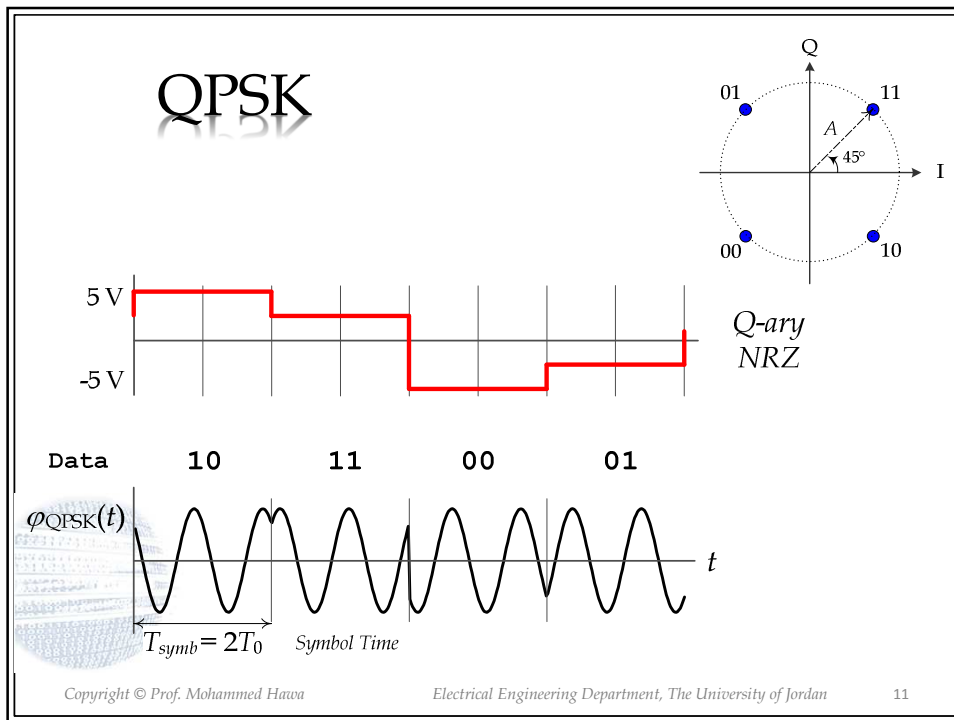
- $m(t)$  is Polar NRZ + **AM** = **ASK**
- $m(t)$  is Polar NRZ + **FM** = **FSK**
- $m(t)$  is Polar NRZ + **PM** = **BPSK**
- $m(t)$  is  $Q$ -ary NRZ + **PM** = **QPSK**
- $m(t)$  is  $M$ -ary NRZ + **PM** = **M-PSK**
- $m(t)$  is  $M$ -ary NRZ + **QAM** = **QAM**
- $m(t)$  is  $M$ -ary NRZ + **AM** = **M-ASK**

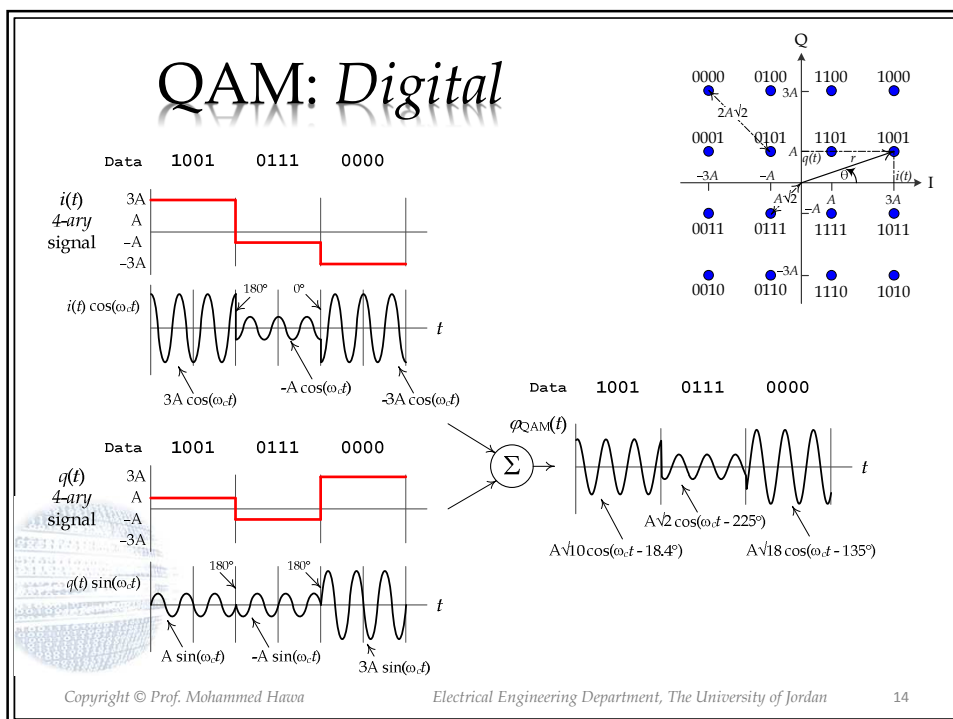
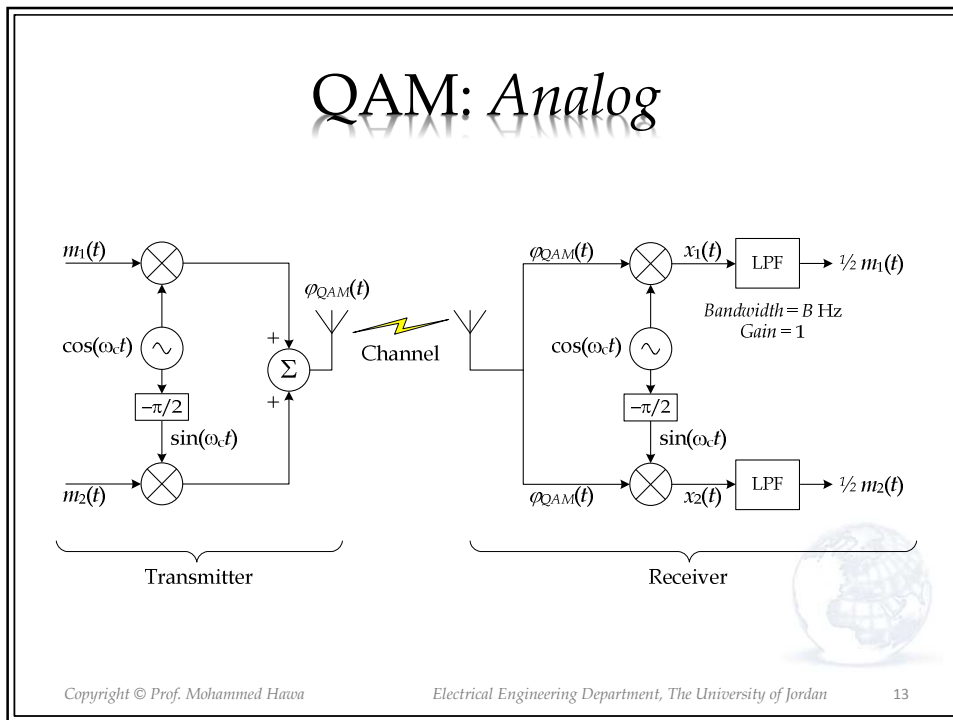




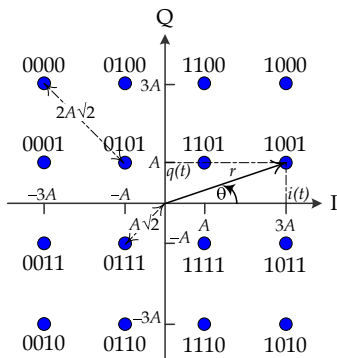








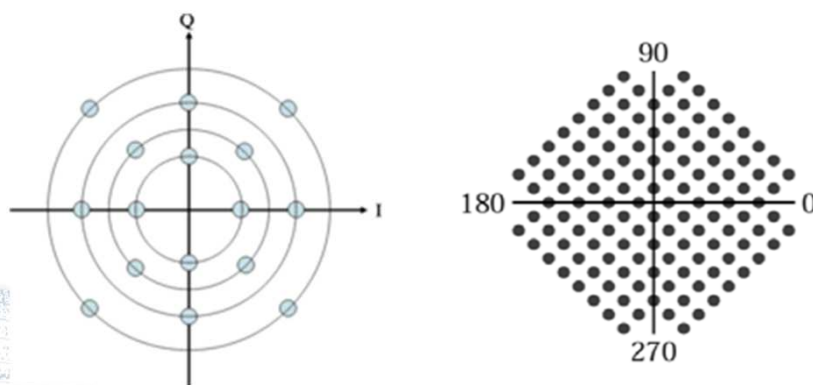
# 16-QAM Constellation Diagram



**Example:** Find the bandwidth of an 16-QAM modulated signal if the data bit rate is 8 Mbit/s.

**Solution:** For 16-QAM, Bandwidth =  $2 \times \text{Baud Rate} = 2 \times \frac{8\text{Mbps}}{\log_2(16)} = 2 \times \frac{8\text{Mbps}}{4 \text{ bits/symb}} = 4 \text{ MHz}$ .

# Many QAM Constellations

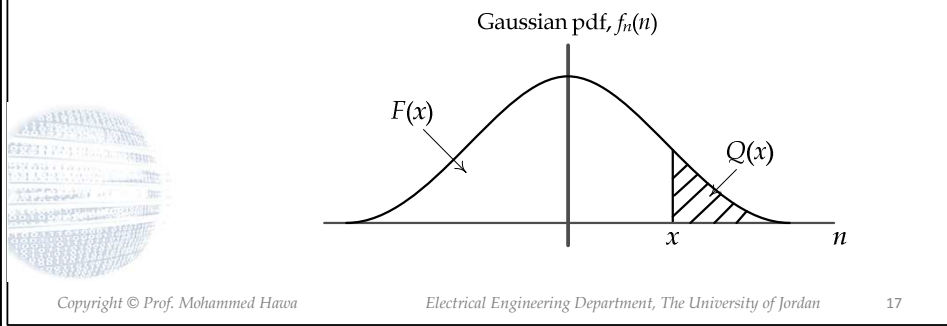




# AWGN Noise

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

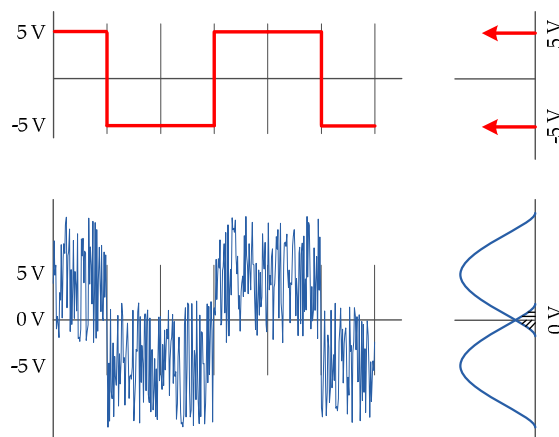
$$Q(x) = 1 - F(x) = 1 - \int_{-\infty}^x f(\alpha) d\alpha = \int_x^{\infty} f(\alpha) d\alpha = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{\alpha^2}{2}} d\alpha$$



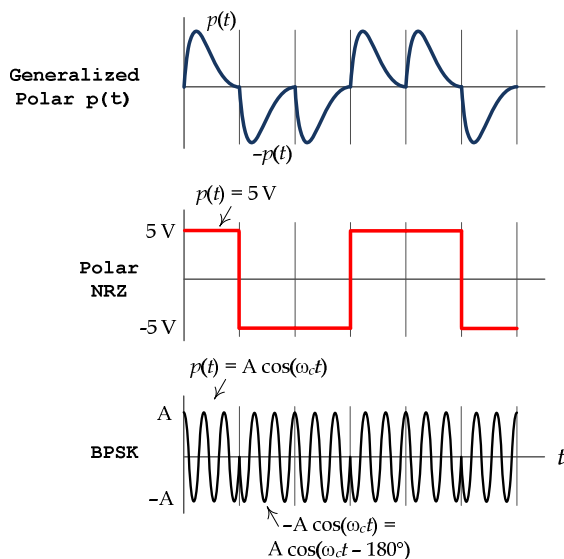
# Quantile Function $Q(x)$

x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)
2.00	2.28E-02	3.00	1.35E-03	4.00	3.17E-05	5.00	2.87E-07
2.05	2.02E-02	3.05	1.14E-03	4.05	2.56E-05	5.05	2.21E-07
2.10	1.79E-02	3.10	9.68E-04	4.10	2.07E-05	5.10	1.70E-07
2.15	1.58E-02	3.15	8.16E-04	4.15	1.66E-05	5.15	1.30E-07
2.20	1.39E-02	3.20	6.87E-04	4.20	1.33E-05	5.20	9.96E-08
2.25	1.22E-02	3.25	5.77E-04	4.25	1.07E-05	5.25	7.60E-08
2.30	1.07E-02	3.30	4.83E-04	4.30	8.54E-06	5.30	5.79E-08
2.35	9.39E-03	3.35	4.04E-04	4.35	6.81E-06	5.35	4.40E-08
2.40	8.20E-03	3.40	3.37E-04	4.40	5.41E-06	5.40	3.33E-08
2.45	7.14E-03	3.45	2.80E-04	4.45	4.29E-06	5.45	2.52E-08
2.50	6.21E-03	3.50	2.33E-04	4.50	3.40E-06	5.50	1.90E-08
2.55	5.39E-03	3.55	1.93E-04	4.55	2.68E-06	5.55	1.43E-08
2.60	4.66E-03	3.60	1.59E-04	4.60	2.11E-06	5.60	1.07E-08
2.65	4.02E-03	3.65	1.31E-04	4.65	1.66E-06	5.65	8.02E-09
2.70	3.47E-03	3.70	1.08E-04	4.70	1.30E-06	5.70	5.99E-09
2.75	2.98E-03	3.75	8.84E-05	4.75	1.02E-06	5.75	4.46E-09
2.80	2.56E-03	3.80	7.23E-05	4.80	7.93E-07	5.80	3.32E-09
2.85	2.19E-03	3.85	5.91E-05	4.85	6.17E-07	5.85	2.46E-09
2.90	1.87E-03	3.90	4.81E-05	4.90	4.79E-07	5.90	1.82E-09
2.95	1.59E-03	3.95	3.91E-05	4.95	3.71E-07	5.95	1.34E-09

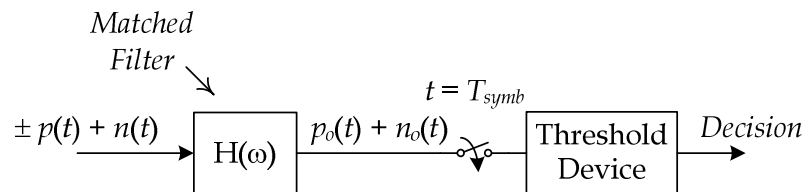
# Why Quantile?



Data 1 0 0 1 1 0



# Matched Filter



# Performance of Digital Systems

Modulation with AWGN	Error Probability
ASK	$BER = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
FSK	$BER = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
BPSK	$BER = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
QPSK	$BER = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
PSK (order M)	$BER \cong \frac{2}{k} Q\left(\sqrt{2k \frac{E_b}{N_0} \times \sin\left(\frac{\pi}{M}\right)}\right)$
QAM (order M) (Rectangular QAM)	$P_{bc} = \frac{4}{k} \left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k}{M-1} \frac{E_b}{N_0}}\right)$ $BER = 1 - (1 - P_{bc})^2$



# Definitions

For the rest of this document, we will use the following notation:

- $M$  = Number of possible symbols that the modulated signal can assume.
- $k$  = the number of bits sent per transmitted symbol =  $\log_2(M)$ .
- $E_s$  = Average energy-per-transmitted-symbol in the modulated signal (Joule).
- $E_b$  = Average energy-per-transmitted-bit in the modulated signal (Joule) =  $E_s/k$ .
- $S_n(\omega) = \frac{N_0}{2}$  = Double-sided noise power spectral density (in W/Hz = Joule).
- $T_o$  = Bit duration.
- $T_{symp}$  = Symbol duration =  $k T_o$
- **BER** = Probability of bit-error = bit error rate.



*Example:*

Find the BER for BPSK if we use an *optimal* detector (a matched filter). Assume the amplitude of the carrier is  $A = 0.5$  V, data rate is 2 bps, and  $N_0 = 2 \times 10^{-2}$  W/Hz.

*Solution:*

In BPSK there is one symbol per bit (i.e., a total of two symbols that the modulated signal can assume). The two symbols can be written as:

$$s_1 = A \cos(\omega_c t) \qquad s_2 = -A \cos(\omega_c t) = A \cos(\omega_c t - \pi)$$

The energy-per-symbol here is the same as the energy-per-bit and is equal for both possible symbols. Hence, its average is:

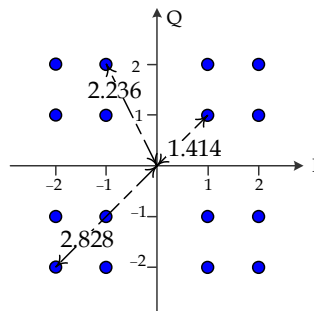
$$E_b = E_s = \left( \frac{A^2}{2} T_{symp} \right) \Pr[1] + \left( \frac{A^2}{2} T_{symp} \right) \Pr[0] = \frac{A^2}{2} T_{symp} = \frac{A^2}{2} T_o = \frac{A^2}{2} \frac{1}{f_0}$$

Hence,

$$\begin{aligned} \text{BER} &= Q \left( \sqrt{\frac{2E_b}{N_0}} \right) = Q \left( \sqrt{\frac{A^2}{N_0 f_0}} \right) = Q \left( \sqrt{\frac{0.5^2}{2 \times 10^{-2} \times 2}} \right) = Q(\sqrt{6.25}) = Q(2.5) \\ &= 6.21 \times 10^{-3} \end{aligned}$$

## Example

Find the BER for the 16-QAM constellation shown below if we use an *optimal* detector (a matched filter). Assume the data rate is 4 bps, and  $N_0 = 5 \times 10^{-2}$  W/Hz.



### Solution:

In this system there are 16 possible symbols, which we assume to be equally probable, i.e., each occurs with a probability of 1/16. Hence, the energy-per-symbol is:

$$E_s = \left( \frac{1.414^2}{2} T_{\text{symp}} \right) \left( \frac{4}{16} \right) + \left( \frac{2.236^2}{2} T_{\text{symp}} \right) \left( \frac{8}{16} \right) + \left( \frac{2.828^2}{2} T_{\text{symp}} \right) \left( \frac{4}{16} \right)$$

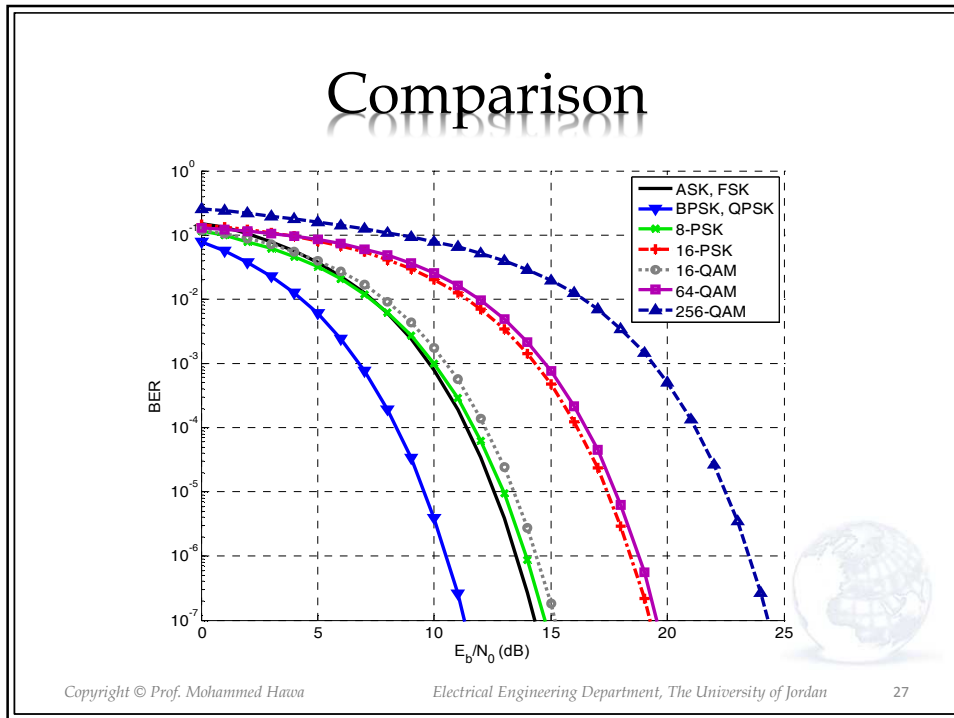
$$E_s = [0.25 + 1.25 + 1] T_{\text{symp}} = 2.5(T_{\text{symp}})$$

$$E_b = \frac{E_s}{k} = 2.5 \left( \frac{T_{\text{symp}}}{k} \right) = 2.5(T_0) = \frac{2.5}{f_0} = \frac{2.5}{4} = 0.625 \text{ J}$$

$$P_{bc} = \frac{4}{k} \left( 1 - \frac{1}{\sqrt{M}} \right) Q \left( \sqrt{\frac{3k}{M-1} \frac{E_b}{N_0}} \right) = \frac{4}{4} \left( 1 - \frac{1}{\sqrt{16}} \right) Q \left( \sqrt{\frac{3 \times 4}{16-1} \times \frac{0.625}{0.05}} \right) = \frac{3}{4} Q(\sqrt{10})$$

$$= \frac{3}{4} Q(3.162) = \frac{3}{4} \times 8 \times 10^{-4} = 6 \times 10^{-4}$$

$$BER = 1 - (1 - P_{bc})^2 = 1 - (1 - 6 \times 10^{-4})^2 = 1.2 \times 10^{-3}$$



## Comparison

Modulation	Bandwidth	Error free $E_b/N_0$ (i.e., $BER < 10^{-6}$ )
ASK	$2f_o$	13.5 dB
FSK	$2\Delta f + 2B = 2f_o(\beta + 1)$	13.5 dB
BPSK	$2 \times \text{Baud} = 2f_o$	10.5 dB
QPSK	$2 \times \text{Baud} = f_o$	10.5 dB
8-PSK	$2 \times \text{Baud} = 2f_o/3$	14 dB
16-PSK	$2 \times \text{Baud} = f_o/2$	18 dB
16-QAM	$2 \times \text{Baud} = f_o/2$	14.5 dB
64-QAM	$2 \times \text{Baud} = f_o/3$	18.5 dB
256-QAM	$2 \times \text{Baud} = f_o/4$	23.4 dB

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## Remember: Digital Modulation

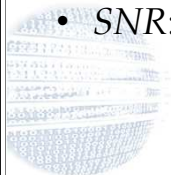
- **Bandwidth** of the channel decides the **baud rate** (symbols per second) you can send.
- **Signal-to-noise ratio** ( $E_b/N_0$ ) decides the level of modulation you can use while still maintaining a small bit error rate. In other words, it decides the number of **bits you can send per symbol**.
- Hence, the two factors together (bandwidth and SNR) decide the **total bit rate** you can achieve over any single channel.
- Shannon's Limit!



## Shannon's Limit

$$C = B_{ch} \times \log_2(1 + SNR)$$

- $C$ : Capacity of the channel in bits/second (bps)
- $B_{ch}$ : Channel bandwidth (units of Hz)
- $SNR$ : Signal-to-Noise Ratio (unitless)(*not dB*)



# Applications

- **IEEE 802.11 (Wi-Fi):** BPSK, QPSK, 16-QAM, 64-QAM and CCK (Complementary Code Keying) (CCK is an extension of QPSK).
- **IEEE 802.16 (Wi-MAX):** BPSK, QPSK, 16-QAM, and 64-QAM. It uses these modulation schemes in combination with OFDM (Orthogonal Frequency division multiplexing) (OFDM is an extension of FDM).
- **DVB (Digital Video Broadcasting):** DVB-S (for satellite broadcasting) uses QPSK or 8-PSK; DVB-C (for cable) uses 16-QAM, 32-QAM, 64-QAM, 128-QAM or 256-QAM; and DVB-T (for terrestrial television broadcasting) uses 16-QAM or 64-QAM.
- **DAB (Digital Audio Broadcasting):** DQPSK (Differential QPSK) (DQPSK is a variation of QPSK).
- **ADSL:** QAM in a scheme called DMT (Discrete Multi-Tone modulation).

